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Mobility and Long term Equality of Opportunity

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Mobility and Long term Equality of Opportunity

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Abstract

The aim of this paper is to propose a methodology for evaluating long-term income distributions according to the equality of opportunity principle; we propose partial and complete rankings of long term income distributions and show the relationship between the inequality of opportunity in the single periods of time and inequality of opportunity in the long run. We show that this relationship can be interpreted in terms of intragenerational mobility. In general, it is possible to state that mobility can act as an equalizer of opportunities when the accounting period is extended.

Keywords: Equality of opportunity, income mobility, inequality, social welfare. **JEL classification:** D71, D91, I32.

1 Introduction

Recent contributions have expressed an increasing discontent with the use of observations of income for a single year in distributional analysis. The reason is twofold: on the one hand, the existence of transitory income components, which may cause inequality in annual income to be systematically higher than long-term income inequality, if idiosyncratic shocks to income average out over time. On the other hand, the life cycle effect: measuring income early (late) in individuals' working lifespan is expected to understate (overstate) long-term income inequality, as individuals with high permanent income tend to be those with high income growth. The combination of these two factors may determine a high degree of mobility in the individual income at different points in time.

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As a consequence, an evaluation based on snapshot income distributions may give a very different picture of an evaluation based on long term distributions.

This is true both if one is interested in measuring inequality and social welfare according to the equality of outcome or according to equality of opportunity (EOp). However, in most of theoretical and empirical works on equality of opportunity snapshots of income still form the basis of analysis (notable exceptions are Bourguignon et al. 2007 and Aaberge et al. 2010). The aim of this paper is to propose a methodology for evaluating long-term income distributions according to the EOp principle; we propose partial and complete rankings of long term income distributions and show the relationship between the inequality of opportunity in the single periods of time and inequality of opportunity in the long run. We show that this relationship can be interpreted in terms of intra-generational mobility. In general, it is possible to state that mobility can act as an equalizer of opportunities when the accounting period is extended.

We refer to the concept of EOp that has been introduced in political philosophy by authors such as Rawls (1971), Dworkin (1981a,b), Sen (1985), and, in particular, by Arneson, (1989) and Cohen (1989). Following this literature, and inspired mainly by Roemer (1993, 1998) and Fleurbaey (1995, 2008), economists have over the last two decades explored different ways in which the concept of EOp may be translated in formal economic models and have proposed different methodologies to measure inequality of opportunity¹.

We follow the EOp literature in assuming that individuals' outcomes arise from two different types of variables: variables which they should not be held responsible for (circumstances), and variables which belong to the sphere of individuals' responsibility (effort). Once this basic partition has been made, the concept of EOp can be decomposed into two distinct ethical principles: the Compensation Principle, which states that differences in outcomes due to circumstances are ethically unacceptable and should be compensated, and the Reward Principle, which states that differences due to effort are to be considered ethically acceptable and do not justify any redistribution.

In our context, there is equality of opportunity if the set of opportunities is the same for all individuals, regardless of their circumstances. Thus, inequality of opportunity is reduced if inequality between individual opportunity sets decreases. This approach partitions the population into different types, where each type is formed by individuals endowed with the same set of circumstances. The type-specific outcome distribution is interpreted as the opportunity set of individuals with the same circumstances. Accordingly, it focuses on inequality between types, and is neutral with respect to inequality within types².

¹On the measurement of opportunity inequality see, among others, Peragine (2002, 2004a, 2004b), Bourguignon et al. (2003), Dardanoni et al. (2006), Lefranc et al. (2006), Ferreira and Guignoux (2008), Peragine and Serlenga (2008) and Checchi and Peragine (2005, 2010).

²This approach is called *ex ante* or *type* approach in Peragine (2002), Fleurbaey and Peragine (2009). This is the approach proposed by Van de Gaer (1993) and Peragine (2004b) and used by Ferreira and Guignoux (2008), Lefranc et al. (2006), Rodriguez (2008). A different approach which has been proposed in the literature is the *ex post* or *tranche* approach. This is the approach proposed by Roemer (1993, 1998), and used by Roemer et al. (2003) and Aaberge and Colombino (2010). Peragine (2002, 2004a) and Checchi and Peragine (2010)

This approach has been formulated in a static context, where current incomes form the basis of the analysis. In this paper, we propose a framework that can be used to measure long-term EOp, and we show how it relates to intra-generational mobility.

As for intra-generational mobility, we start from the notion that the state of no mobility is assumed to occur when each individual does not undertake any reranking in the income distributions in different periods of time (Aaberge et al., 2002; Aaberge and Mogstad, 2010). On this baseline we derive an index of mobility, which deals with distribution of sets of opportunities. In this vein, we identify an index of intragenerational mobility, with a clear interpretation within the conceptual framework of equality of opportunity.

Furthermore, we propose a normative interpretation of the index. In fact, by using a rank dependent concept of social welfare, we are able to work out an ethical index of mobility, which will help us to shed light on the normative implications of mobility. That is, to understand whether higher mobility can be defined as welfare improving.

The work is structured as follows. In Section 2 the general EOp framework will be presented, followed by the derivation of our index of mobility; in Section 3 the normative implications will be described; Section 4 concludes.

2 The framework

We consider a population of N individuals, each of them holding a certain amount of income, X , and we have T periods of time. The individual income is function of two main components: the set of circumstances, c , belonging to a finite set $\Omega = \{c_1, \dots, c_s\}$, and the level of effort, $e_t \in \Theta \subseteq \mathbb{R}_+$. The individual cannot be held responsible for c , which is fixed over time; he is, instead, responsible for the effort he autonomously decides to exert in every period of time, thus e is dependent of time. The model is defined by:

$$X_t = f(c, e_t), \forall t \in \{1, \dots, T\} \quad (2.1)$$

Where $f = \Omega \times \Theta \rightarrow \mathbb{R}_+$ is assumed to be continuous and monotonic in e and it is the same for the whole population, whose income distribution is defined by a cumulative distribution function $F : \mathbb{R}_+ \rightarrow [0, 1]$.

We assume that we can identify s subgroups of the population, according to the set of circumstances characterizing each individual. A "type" is the label used to identify each subgroup. Therefore, when we refer to type i , we mean all the individuals with the set of circumstances c_i .

explore both the ex ante and the ex post approaches. See also Ruiz-Castillo (2004) for an analysis of mobility in the context of inequality of opportunity.

Given the aim of this paper, it is necessary to evaluate lifelong individual income streams, through a measure of permanent income. For the sake of exposition we make some simplifying assumptions³.

Let X_{it} be a random variable, denoting income of individual i at time t .

Assumption 1. Rate of interest equal to the rate of time preference, and both equal to 0.

Assumption 2. Absence of credit market imperfections.

As demonstrated by Aaberge and Mogstad (2010), assumption 1 and 2 justify the employment of the average income across time as a measure of permanent income, i.e.:

$$X_{i+} = \frac{1}{T} \sum_{t=1}^T X_{it}, \quad i=1, \dots, N$$

The two distributions we will deal with are:

(X_{1t}, \dots, X_{Nt}) : distribution of individual incomes at time t ;

(X_{1+}, \dots, X_{N+}) : distribution of individual permanent incomes.

Let the overall mean income across individuals at time t , $X_{+t} = \frac{1}{N} \sum_{i=1}^N X_{it}$

$t = 1, 2, \dots, T$, and let μ_{+t} , where $\mu_{+t} = E(X_{+t})$ and $\mu = \sum_{t=1}^T \mu_{+t}$.

Two additional assumptions of our model are the following.

Assumption 3. There is a normative agreement on the appropriate list of circumstances.

Assumption 4. The set of initial circumstances is constant over time.

2.1 Lorenz opportunity partial orderings

A society shows equality of opportunity, when it can grant to each individual the same set of opportunities independently of the conditions acquired at birth, since he cannot be held responsible for them.

Let \bar{X}_j be the vector of permanent incomes for type j . The overall distribution of permanent incomes partitioned by type is:

$$(\bar{X}_1, \dots, \bar{X}_j, \dots, \bar{X}_s)$$

Substituting the income of those individuals belonging to the same type with the mean income of that type, we get the distribution of type mean incomes (μ_j is the permanent mean income of type j); it has the advantage of eliminating the inequality within type:

$$X_a = (\mu_1 1_{N_1}, \dots, \mu_j 1_{N_j}, \dots, \mu_s 1_{N_s})$$

³See Aaberge *et al.* (2010) for a detailed derivation of a measure of permanent income in line with the EOP framework.

To keep the exposition simple we assume that the types have all the same population⁴.

We assume that types' mean permanent incomes are distributed in ascending order: $\mu_1 \leq \mu_2 \leq \dots \leq \mu_s$, such that the mean income of type j is lower than the mean income of type⁵ $j + 1$, for all $j = 1, \dots, s - 1$.

The opportunity Lorenz curve expressing inequality between types is defined by⁶:

$$L_a \left(\frac{j}{s} \right) = \frac{\sum_{k=1}^j \mu_k}{\sum_{k=1}^s \mu_k}, \quad j = 1, \dots, s \quad (2.2)$$

In the context of our analysis, between types inequality can be interpreted as a form of inequality of opportunity, hence, the opportunity Lorenz curve provides a partial dominance condition, that can be used to order distributions according to the amount of inequality of opportunity they show.

A different representation of (2.2) can be derived using the overall type mean income:

$$\mu_{X_a} = \frac{1}{s} \sum_{k=1}^s \mu_k \quad (2.3)$$

Inserting (2.3) in (2.2) yields:

$$L_a \left(\frac{j}{s} \right) = \frac{\sum_{k=1}^j \mu_k}{s \mu_{X_a}}, \quad j = 1, \dots, s \quad (2.4)$$

In order for the information provided by (2.4) to be effectively used in terms of mobility measurement, we need to define the Lorenz curve for the type mean income distribution in each period t .

Let X_{jt} be the vector of income for type j , at time t . The overall distribution of incomes at time t partitioned by type is:

$$(X_{1t}, \dots, X_{jt}, \dots, X_{st}), \quad t = 1, \dots, T.$$

We substitute individuals' income in the same type with the mean income, at time t , of the type they belong to, this yields the type mean income distribution at time t :

$$X_{at} = (\mu_{1t} 1_{N_1}, \dots, \mu_{jt} 1_{N_j}, \dots, \mu_{st} 1_{N_s}) \quad t = 1, \dots, T.$$

⁴This can look as a strong assumption; note however that the results derived in the paper are also valid in general, when we remove this assumption.

⁵In what follows we will extend to the EOp scenario the technique introduced by Aaberge and Mogstad (2010) for the measurement of income mobility.

⁶See Peragine (2002, 2004b) for characterizations of inequality of opportunity partial orderings based on the Lorenz dominance.

We assume that the type mean incomes at time t are distributed in ascending order $\mu_{1t} \leq \dots \leq \mu_{st}$ ⁷.

The opportunity Lorenz curve for X_{at} is defined by:

$$L_{at} \left(\frac{j}{s} \right) = \frac{\sum_{k=1}^j \mu_{kt}}{\sum_{k=1}^s \mu_{kt}}, j = 1, \dots, s; t = 1, \dots, T \quad (2.5)$$

Similar as above, the overall mean of the type mean income distribution at time t is given by:

$$\mu_{X_{at}} = \frac{1}{s} \sum_{k=1}^s \mu_{kt}, t = 1, \dots, T \quad (2.6)$$

From which:

$$L_{at} \left(\frac{j}{s} \right) = \frac{\sum_{k=1}^j \mu_{kt}}{s \mu_{X_{at}}}, j = 1, \dots, s; t = 1, \dots, T \quad (2.7)$$

Hence, we have

$$s \mu_{X_{at}} L_{at} \left(\frac{j}{s} \right) = \sum_{k=1}^j \mu_{kt}, j = 1, \dots, s; t = 1, \dots, T$$

and

$$s \mu_{X_a} L_a \left(\frac{j}{s} \right) = \sum_{k=1}^j \mu_k, j = 1, \dots, s$$

We can now establish a relationship between (2.4) and (2.7), representing the inequality between types respectively for the long term and short term distributions. We observe that if each type keeps the same position along the type mean income scale over time, then⁸ the Lorenz curve of the permanent

⁷As before, we do not consider the population size of each type.

⁸This procedure has been suggested by Aaberge *et al.* (2002) and by Aaberge and Mogstad (2010) for measuring inequality of outcome.

distribution can be decomposed in terms of the time specific opportunity Lorenz curves of the same distribution at each time t . Under the assumption that the type position in the mean income distribution is constant over time.

$$L_a \left(\frac{j}{s} \right) = \frac{\sum_{k=1}^j \mu_k}{s\mu_{X_a}} = \frac{\sum_{t=1}^T \sum_{k=1}^j \mu_{kt}}{s\mu_{X_a}} = \sum_{t=1}^T \frac{\mu_{X_{at}}}{\mu_{X_a}} L_{at} \left(\frac{j}{s} \right) \quad (2.8)$$

As demonstrated in (2.8) when each type position in X_{at} is unaltered over time, the opportunity Lorenz curve of X_a is equivalent to a weighted average of the opportunity Lorenz curve for the distribution X_{at} , $t = 1, \dots, T$.

Eq. (2.8) has an interesting interpretation in the framework we propose. In fact, it states that, according to our model, inequality of opportunity in the long run can be expressed as a weighted average of inequality of opportunity in the snapshot income distributions. Therefore, a first result of our work is that, by extending the analysis of inequality of opportunity to the long run, which solves many of the problems that a static approach would have, we are able to explain the relationship existing between short run and long run inequality of opportunity.

2.1.1 Gini index of inequality of opportunity and mobility

In the previous subsection we have provided a decomposition of the Lorenz curve, defined for the permanent type mean income distribution, in the Lorenz curve defined for the type mean income distribution at each period of time. However, only a partial ordering is possible by adopting the Lorenz curve as dominance criterion. Thus, it is necessary to discern, among the inequality indices, the one which is consistent with the Lorenz dominance. The literature on income inequality is developed in this field, and numerous studies provide a justification for the adoption of the Gini coefficient, as a measure summarizing the information provided by the Lorenz curve in terms of inequality⁹. Therefore, it appears natural to employ the same coefficient, which we compute on X_a and X_{at} and denote respectively by G_a and G_{at} . As it is applied on distributions, which, by construction, eliminate the inequality within type, the kind of inequality captured by this index is an inequality between types, or inequality of opportunities. Let the Gini index of long term inequality of opportunity, G_a , be defined by:

$$G_a = \sum_{k=1}^s \sum_{j=1}^s \frac{|\mu_k - \mu_j|}{2s^2 \mu_{X_a}}$$

Let the Gini index of short term inequality of opportunity, G_{at} , be defined by:

⁹See Aaberge (2000, 2001), Lambert (2001).

$$G_{at} = \sum_{k=1}^s \sum_{j=1}^s \frac{|\mu_{kt} - \mu_{jt}|}{2s^2 \mu_{X_a t}}$$

Under the assumption that there is not reranking among the types, and using the above expression of G_a and G_{at} , we can show that a relationship can also be established between G_a and G_{at} .

$$\begin{aligned} G_a &= \sum_{k=1}^s \sum_{j=1}^s \frac{|\mu_k - \mu_j|}{2s^2 \mu_{X_a}} = \sum_{k=1}^s \sum_{j=1}^s \sum_{t=1}^T \frac{|\mu_{kt} - \mu_{jt}|}{2s^2 \mu_{X_a}} = \sum_{t=1}^T \frac{2s^2 \mu_{X_a t} G_{at}}{2s^2 \mu_{X_a}} = \\ &= \sum_{t=1}^T \frac{\mu_{X_a t}}{\mu_{X_a}} G_{at} \end{aligned} \quad (2.9)$$

Expression (2.9) tells us that G_a is equivalent to a weighted average of G_{at} .

As explained for the opportunity Lorenz curve, in the current scenario, between type inequality represents a form of inequality of opportunity. Hence, the Gini coefficient, being a synthetic index, provides a complete dominance condition, that can be used to rank distributions on the basis of opportunity inequality, both in the long run and in the short run.

Again, we are able to explain a possible relationship between long run and short run inequality of opportunity. For this relation to hold, the condition is that each type j keeps the same position in X_{at} , every period t , that is, if there is no reranking. Now, we can suppose that a state of no mobility is, in fact, verified when no one moves from its original position. In line with our framework, we can define immobility as a situation where each type, and with it the individuals it represents, remains attached to the same rank in every period. Thus, (2.9) proves to be an appropriate benchmark for mobility measurement; every variation from that value should be interpreted as symptom of mobility. It follows that a measure of mobility can be defined by:

$$M_a = \frac{\sum_{t=1}^T \frac{\mu_{X_a t}}{\mu_{X_a}} G_{at} - G_a}{\sum_{t=1}^T \frac{\mu_{X_a t}}{\mu_{X_a}} G_{at}} \quad (2.10)$$

or equivalently

$$M_a = 1 - \frac{G_a}{\sum_{t=1}^T \frac{\mu_{X_B t}}{\mu_{X_B}} G_{at}} \quad (2.11)$$

This is a measure of intragenerational mobility, coherent with the principle of reward. Clearly, this index allows for very intuitive information. Firstly, being¹⁰ $0 \leq M \leq 1$, the results are rather intuitive. A value of the index equal to 0 means that no mobility has taken place; on the contrary, the closer the index to 1, the more mobility has exerted its effect in order to equalize X_a over time. Mobility is maximized if the process has been successful in generating a perfect equalization of opportunity in the long term, a situation corresponding to a value of $G_a \rightarrow 0$, as compared to its aggregation over time. The index provides us information about the extent of the reranking in the type mean income distribution, that is how much types interchange their position in the income parade, over time. Consider two distributions: F_A and F_B , with same G_a , but with a higher mobility for F_A ; this means that either the process of type reranking has been more effective in equalizing opportunity for F_A , or there has been more reranking in F_A as compared to F_B . However, an higher mobility can be due to the fact that the time specific distributions associated with F_A exhibit higher inequality; therefore, if F_A and F_B show the same level of inequality in the permanent distribution, this means that mobility has been successful in attenuating short terms differentials in the income distribution.

As a result, M_a measures how much the inequality between types can be reduced by mobility, if we extend the accounting period. This implies that mobility can effectively act as an equalizer of opportunities over time. In fact, this index provides a measure of the extent to which the types have interchanged their position, when we extend the time span. Therefore, in the long term, there is more equality of opportunity if we allow the individuals belonging to each type changing their rank, that is, interchanging their set of opportunities every period. An important implication of adopting this methodology concerns the ordering of distributions. It is possible to state that, even if we cannot order distributions in the short term, because they show the same level of inequality of opportunity, a distribution of type mean income can dominate another distribution when it shows a higher level of mobility.

In the light of the theoretical principles underling the opportunity egalitarian framework, we can grasp the intuition behind the implementation of this index, as compared with the traditional measures of income mobility. In fact, if we consider a general measure of income mobility, it is not always properly right to infer that income mobility is welfare improving, since it could be the case that mobility within types prevails, which is not desirable, being due to a variation in the level of effort. On the contrary, (2.11) allows having information on the extent of mobility between types, which reflects the kind of inequality judged as unfair by the society. In this perspective, this index could be computed as an instrument to investigate the effectiveness of policy interventions, aimed at reducing inequality due to different sets of circumstances, as well.

The insights stemming from this index not only justify, but requires equality of opportunity to be evaluated for long term distributions of income, since,

¹⁰This is because $G_a \leq \sum_{t=1}^T \frac{\mu_{X_a,t}}{\mu_{X_a}} G_{at}$

mobility may act reducing the opportunity inequality. Moreover, the possibility of measuring income mobility in this framework gives new relevance to the use of the Gini coefficient, in addition to the subgroup decomposable inequality measures, typically adopted in this framework. We are aware of the drawbacks related to the use of the Gini coefficient in the EOp measurement. In fact, the adoption of this index does not allow for unambiguous decomposition of the overall inequality in opportunity inequality and effort inequality (see Checchi and Peragine, 2010)¹¹, nevertheless, the Gini coefficient appears to be the most appropriate for the analysis we propose in this paper, since our aim is different and we are concerned with aggregation over time.

Hence, an additional result of our work is that we provide an approach to measure EOp in the long term and to compare the differences existing between long term and short term inequality of opportunity. Finally, we are able to explain these differences with a measure of intra-generational mobility in the perspective of the EOp theory.

2.1.2 A general family of rank dependent indices of inequality of opportunity and mobility

Employing as baseline the distributions derived above, X_a and X_{at} , we propose an index of mobility based on rank dependent inequality measures¹². A rank dependent measure of long term inequality between types is (see Aaberge *et al.*, 2010):

$$J_a = 1 - \frac{\sum_{j=1}^s p_j \mu_j}{\mu_{X_a} \sum_{j=1}^s p_j} \quad (2.12)$$

where p is a weight function expressing the normative judgement of a social planner on the reference distribution; it depends on the position of each type in the type mean-income distribution. This implies that p_j is sensitive only to inequality between types caused by different sets of initial circumstances, but neutral to differences in the final outcome due to the effort exerted.

It follows that: $p_j \geq 0, j = 1, \dots, s$, since income is positively valued, no matter the position in the income ranking of the type owning that income. In addition, a social decision maker, who agrees on some egalitarian principles, should be adverse to opportunity inequality, a behavior arising when the weights are non-increasing, i.e. $p_1 \geq p_2 \geq \dots \geq p_s$, that is, $p'_j \leq 0$. This implies that higher weight is given to those who suffer from bad opportunities. This is in line with the principle underlying the *ex-ante* approach, which is neutral to inequality of outcome due to different levels of effort exerted, but requires

¹¹Note, however, that Aaberge *et al.* (2010) provide a decomposition of overall inequality into inequality of outcome and inequality of opportunity for the *ex-post* approach.

¹²See Donaldson and Weymark (1980, 1983), Aaberge (2000, 2001), for an extensive discussion on rank dependent measures of inequality.

that the outcome is the same, independently from the set of circumstances. Therefore, the weights are sensitive to the type rank and neutral to the effort rank.

Expression (2.12) preserves first-degree¹³ Lorenz dominance. Since it is expressed over a distribution embodying the reward principle, it describes the extent of the rank dependent inequality due to initial factors. It captures the part of inequality due to unfairness, taking into account the importance that the society attaches to the social ranking.

We can show that, in the absence of reranking, the equivalence between the measures of inequality of opportunity, concerning the permanent distribution of income and the distribution at time t , holds also in presence of rank dependent measures of inequality of opportunity¹⁴.

At time t , the rank dependent measure of inequality of opportunity is defined

by:

$$J_{at} = 1 - \frac{\sum_{j=1}^s p_j \mu_{jt}}{\mu_{X_{at}} \sum_{j=1}^s p_j} \quad (2.13)$$

Combining (2.12) and (2.13):

$$\begin{aligned} \mu_{X_a} J_a &= \mu_{X_a} - \frac{\sum_{j=1}^s p_j \mu_j}{\sum_{j=1}^s p_j} = \sum_{t=1}^T \mu_{X_{at}} - \frac{\sum_{j=1}^s \sum_{t=1}^T p_j \mu_{jt}}{\sum_{j=1}^s p_j} = \\ &= \sum_{t=1}^T \left(\mu_{X_{at}} - \frac{\sum_{j=1}^s p_j \mu_{jt}}{\sum_{j=1}^s p_j} \right) = \sum_{t=1}^T \mu_{X_{at}} J_{at} \end{aligned}$$

Hence, we have:

$$J_a = \sum_{t=1}^T \frac{\mu_{X_{at}}}{\mu_{X_a}} J_{at} \quad (2.14)$$

¹³See Aaberge (2009) for different degrees of Lorenz dominance.

¹⁴In a similar fashion Aaberge and Mogstad (2010) show that the same relationship holds for rank dependent inequality measures applied to distributions of outcome.

In the context of our framework, between type inequality represents a form of inequality of opportunity. The rank dependent measure of inequality provided in eq. (2.12) and (2.13) can be employed to order distributions according to the amount of opportunity inequality they generate, respectively in the long run and in the short run. Hence, eq. (2.14) states that, according to our model, rank dependent inequality of opportunity in the long run can be expressed as a weighted average of rank dependent inequality of opportunity in the snapshot income distributions. Therefore, another result of our work is that, we are able to explain the relationship existing between short run and long run inequality of opportunity, when there is no intra-generational mobility, even adopting a rank dependent measure of inequality of opportunity.

We can now define a rank dependent measure of mobility:

$$M_a = \frac{\sum_{t=1}^T \frac{\mu_{X_a t}}{\mu_{X_a}} J_{at} - J_a}{\sum_{t=1}^T \frac{\mu_{X_a t}}{\mu_{X_a}} J_{at}} \quad (2.15)$$

Eq. (2.15) provides a measure of rank dependent intra-generational mobility, coherent with the principle of reward. When the index is equal to 0 means that no mobility has taken place; on the contrary, the closer the index to 1, the more mobility has exerted its effect in order to equalize X_a over time. The index allows to quantify the extent of the reranking in the type mean income distribution, where the focus is on individuals characterized by worst circumstances.

Finally, eq. (2.15) represents another result of our work. In fact, we provide an approach to measure EOp in the long term, giving more relevance to disadvantaged types. Furthermore, we are able to state that there can be relevant differences between long term and short term inequality of opportunity that can be explained by intra-generational mobility. This statement gives relevance to the need of extending traditional analysis of EOp to the dynamic context, since mobility might act to alleviate the inequality of opportunity arising from snapshot incomes.

3 Normative implications

In this section we discuss the normative implications of mobility. In particular, we use a rank dependent measure of social welfare and we work out a useful decomposition of the mobility index such that we can check whether mobility may be considered welfare improving.

It is widespread in the literature the perception of social welfare as a trade-off between equality and efficiency¹⁵, which arises to be meaningful in terms of complete ordering of distributions. Different contributions (Lambert, 2001; Aaberge,

¹⁵See Lambert (2001) for an extensive discussion on this topic.

2001) show that social welfare admits a decomposition with respect to average income and inequality. Yaari (1988) provides a similar decomposition using members of the family of rank-dependent inequality measures, and a rank dependent expression of social welfare, which can be expressed as $W = \mu(1 - \tilde{J}(L))$, where $\tilde{J}(L)$ is the rank dependent measure of outcome inequality. It turns out that the Yaari social welfare function (YSWF) over income distributions is represented by a weighted average of ordered incomes, where each income is weighted according to its position in the ranking.

A similar formulation for social welfare can be derived under the light of the principles of opportunity egalitarianism. Given our definition of mobility, and the relevance associated to the individual social rank, we agree that this condition must be reflected in the evaluation function we use for measuring social welfare.

Thus, following Yaari's approach, a social welfare function reflecting concern toward the average level of income, aversion to inequality of opportunity, and concern toward social rank, can be expressed by the following:

$$W_a = \mu_{X_a} (1 - J_a) \quad (3.1)$$

Which is equivalent to:

$$W_a = \frac{\sum_{j=1}^s p_j \mu_j}{\sum_{j=1}^s p_j} \quad (3.2)$$

In the EOp scenario, the YSWF over type mean income distributions is expressed as a weighted average of ordered mean incomes, where each type-mean income is weighted according to its position in the rank. Hence, $p_j \forall j = 1, \dots, s$, are the possible different social weights given to different types. Different value judgments are expressed in this framework by selecting different classes of 'social weight' functions and the weights are type (i.e., circumstances) specific. First, we assume that any income increment does not decrease social welfare: $p_j \geq 0$. Second, a social decision maker, who agrees on some egalitarian principles, should be adverse to opportunity inequality, expressed in this case by the inequality between types, a behavior arising when $p_1 \geq \dots \geq p_j \geq \dots \geq p_s$. This condition can be interpreted as the Principle of Transfer between types¹⁶. That is, any YSWF satisfying this condition will not decrease after a transfer of a positive fraction of income, $\varepsilon \geq 0$, from type $j + 1$ to type j , which leaves relative positions unaltered. Eq. (3.2) is consistent with the ex-ante approach, in fact, as described above, the weights depend on the position of each type in the type mean income distribution, and are sensitive only to inequality between types caused by different sets of initial circumstances, but neutral to differences

¹⁶This characterization of the weighting function is consistent with the monotonicity and responsibility properties, in Peragine (2002).

in the final outcome due to the effort exerted. Formulating, again, an analogue of the Pigou–Dalton transfer principle applied to the current context, we have that, the transfer of a small amount of income, from type $j + 1$ to type j , does not decrease social welfare.

The normative justification of (3.2) was proposed by Yaari (1988) as a theoretical approach for ranking distribution functions, and by Ebert (1987) as a value judgement of the trade-off between mean and inequality in deriving social welfare functions, as in (3.1). A mean-independent ordering of income distributions in terms of inequality, forms the basis of Ebert’s (1987) and Aaberge (2001) approach.

To address the question of ranking social states according to the degree of mobility they show, instead of expressing the opportunity egalitarian aim with the measure of mobility proposed in the previous paragraphs, we employ the YSWF. Therefore, we formulate the problem of ranking income distributions according to the EOp theory, expressing the YSWF as function of mobility, which is sensitive with respect to circumstances-based outcome inequalities, but neutral with regard to effort based inequalities.

Thus, we have good reasons to adopt (3.2) in order to get our social welfare based measure of mobility¹⁷. Using (3.2), we explicit J_a as a function of W_a :

$$J_a = 1 - \frac{W_a}{\mu_{X_a}} \quad (3.3)$$

By inserting (3.2) in (2.12) we get:

$$\begin{aligned}
 M_a &= \frac{\sum_{t=1}^T \frac{\mu_{X_{at}}}{\mu_{X_a}} \left(1 - \frac{W_{at}}{\mu_{X_{at}}}\right) - \left(1 - \frac{W_a}{\mu_{X_a}}\right)}{\sum_{t=1}^T \frac{\mu_{X_{at}}}{\mu_{X_a}} \left(1 - \frac{W_{at}}{\mu_{X_{at}}}\right)} = \frac{1 - \sum_{t=1}^T \frac{W_{at}}{\mu_{X_{at}}} - 1 + \frac{W_a}{\mu_{X_a}}}{1 - \sum_{t=1}^T \frac{W_{at}}{\mu_{X_a}}} = \\
 &= \frac{\frac{1}{\mu_{X_a}} \left(W_a - \sum_{t=1}^T W_{at} \right)}{\frac{\mu_{X_a} - \sum_{t=1}^T W_{at}}{\mu_{X_a}}} = \frac{W_a - \sum_{t=1}^T W_{at}}{\mu_{X_a} - \sum_{t=1}^T W_{at}}.
 \end{aligned}$$

Hence:

¹⁷It is important to stress that the YSWF we are dealing with is not based on individual income, but on the type mean-income, thus, it is a summarized way of accounting for social welfare based on opportunity inequality, where the welfare is increasing when opportunity inequality is reduced.

$$M_a = \frac{W_a - \sum_{t=1}^T W_{at}}{\mu_{X_a} - \sum_{t=1}^T W_{at}} \quad (3.4)$$

From eq. (3.4) it is possible to notice that there may arise differences between long term and short term rank dependent social welfare. These differences can be explained by mobility, as showed by the numerator, where $\sum_{t=1}^T W_{at}$ is the social welfare we would have in the absence of reranking. Hence, mobility can be expressed as function of a rank dependent and inequality adverse social welfare. In particular the index in eq. (3.4) is determined by $W_a - \sum_{t=1}^T W_{at}$ which measures the gain in social welfare due to mobility. Also in this case this index goes to 0 when there is no reranking and it is equal to 1 when mobility acts to completely equalize opportunity in the long term. This is the case in which W_a would be equal to μ_{X_a} , where μ_{X_a} is the mean income each type would have when opportunity are fully equalized.

Inverting this equation, it is possible to get a measure of long term social welfare as function of our measure of mobility, which is the EOp version of the decomposition proposed by Aaberge and Mogstad (2010):

$$W_a = M_a \left(\mu_{X_a} - \sum_{t=1}^T W_{at} \right) + \sum_{t=1}^T W_{at} \quad (3.5)$$

Eq. (3.5) allows to obtain a normative interpretation of mobility. In sum, social welfare turns out to be determined by two factors. The first is the extent of mobility, in the period considered, weighted by the maximum gain in social welfare due to mobility. The second is the level of social welfare, in the long run, which would arise in the absence of reranking. Given the positivity of the weight associated to the mobility component, social welfare in the presence of mobility comes out to be higher than social welfare in the absence of mobility. As a result, we can state that mobility is welfare improving, since it affects positively long term social welfare and since it may act to equalize opportunities in the long run.

4 Conclusions

Recent contributions have shown the importance of extending standard distributional analysis to a dynamic context, in order to overcome some of the problems encountered when focusing on snapshots income, such as, idiosyncratic shocks

and life cycle effects. This is true both for the measurement of inequality and social welfare according to the equality of outcome or according to the equality of opportunity. However, in most of theoretical and empirical works on equality of opportunity, snapshots of income still form the basis of the analysis (notable exceptions are Bourguignon et al. 2007 and Aaberge et al. 2010).

In this paper, we have proposed a framework for the measurement of long-term EOp, and we have shown how it relates to intra-generational mobility. Our framework provides to be a satisfactory tool to explain, through mobility, possible differences arising from the comparison between long term and short term inequality of opportunity. From this analysis we have obtained an index of intragenerational mobility with a clear interpretation in the perspective of the EOp theory. Furthermore, our analysis sheds light on the relevance of extending the evaluation of the EOp in the long term. In fact, we have been able to explain the gap between long term and short term inequality of opportunity through mobility. Finally, we have also provided a normative interpretation of mobility. Mobility has been shown to capture the equalization of opportunity due to the reshuffling of individuals, with different circumstances, in the income parade, occurring when we extend the accounting period. Therefore, it can be interpreted as a measure of the amount of exchange mobility under the light of EOp.

We have developed this analysis using the ex ante approach to equality of opportunity; however, a similar procedure, but with different interpretations, can be used for the ex post approach, which captures distinct but relevant, and sometimes conflicting, principles of the EOp theory. This will be the object of future research.

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