History dependent growth incidence: a characterization and an application to the economic crisis in Italy*

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Abstract

The aim of this paper is to propose an axiomatic characterization of an aggregate measure of growth that takes into account the initial economic conditions of individuals, through their position in the initial income distribution. The contribution of our work to the existing literature is twofold. The first is to provide a unifying framework for the derivation of an absolute and a relative measure of individual growth. The second is represented by the aggregation procedure which leads to a generalization of existing measures of growth. We apply our theoretical framework to evaluate the growth processes experienced by the Italian population in the last decade in order to investigate the history dependent distributional effect of the recent economic crisis.

Keywords: Individual income growth, pro-poor growth, economic crisis.

JEL codes: D31, D63, I32.

1 Introduction

Eventful days, such as the different phases of the recent economic crisis rapidly follow each other. These events have motivated a renewed and increasing interest, both among economists and policy makers, in the issue of the measurement of growth and its distributional implications.

We take a history dependent perspective, which evaluates a growth process on the basis of individuals' growth experiences and their position in the initial distribution of income. Approaches taking the latter into account are becoming increasingly popular (Grimm, 2007; Van Kerm, 2009; Bourguignon, 2011; Jenkins and Van Kerm, 2011; Palmisano and Peragine, 2012). Their main

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tool is the "Non-anonymous Growth Incidence Curve" (see Bourguignon 2011) or the "Mobility Profile" (due to its similarity with the measurement of directional income mobility, Van Kerm, 2009), which plots the growth in mean income achieved by those individuals belonging to the same quantile in the initial distribution of income as a function of their quantile in this initial distribution. These contributions provide formal derivations of dominance conditions that can be used to obtain incomplete rankings of growth processes, but, with the exception of Jenkins and Van Kerm (2011), they do not propose a synthetic index of history dependent growth.

The history dependent perspective is different from the pro-poor perspective, which looks at the extent to which poverty declines over time. The main instrument in this literature is the "Growth Incidence Curve" which plots the growth in mean income at the same percentile in the income distributions in two consecutive periods as a function of this percentile (Ravallion and Chen, 2003; Son 2004). Here, contrary to the history dependent perspective, incomes of different individuals are used to compute the growth in mean incomes. A variety of indices for the measurement of pro-poor growth have been proposed (see Gosse et al., 2008; Kakwani and Son, 2008; Kraay, 2006; Kakwani and Pernia, 2000; Essama-Nssah, 2005; Essama-Nssah and Lambert, 2009).

We characterize and apply a measure of history dependent economic growth that aggregates individuals' growth experiences. Our index is expressed as a weighted average of individual growth measures whose weights are decreasing with the rank in the initial distribution of income. The weights turn out to be the weights in the standard single-series Gini (Donaldson and Weymark, 1980), adjusted for the possibility that, starting from the same initial income level, individuals might experience different income growth[1]. We show that this index is closely related to the proposal by Jenkins and Van Kerm (2011), and like their index, it is additively decomposable into a progressivity index, measuring the pure redistributive effect of the growth process, and the average of individual growths.

Hence, we propose an axiomatic characterization of an aggregate measure of history dependent growth, that is, an approach taking into account the inequalities in the initial distribution of income. More in particular, the crucial History Dependent Growth Incidence (HDGI) axiom says that we like redistributions of individual growth in favor of those having a low level of initial income, and are indifferent between growth redistributions among individuals having the same initial level of income. Moreover, from a formal point of view, the contribution of our work to the existing literature is twofold. The first is that we provide a unifying framework for the derivation of an absolute and a relative measure of individual growth. The second is the aggregation procedure which leads to a generalization of existing measures of growth.

With regard to the first aspect, we axiomatize two directional measures of income growth. Both measures satisfy Normalization, Monotonicity and Independence. Normalization (N) and Monotonicity (M) are common properties in the literature: the former implies that the index is equal to 0 if the initial and final level of income are the same; the latter implies that growth is increasing in second period incomes. The Independence condition (IND) is a new property in this literature. It requires that adding a given amount of income to two individuals with the same initial level of income (but possibly different levels of second period incomes) affects their individual growth rates by the same amount. This independence condition is natural in the present context, as it implies that different effects of additions to second period income on individual growth can only be due to differences in initial income levels. Moreover, it enables us to obtain a unifying characterization for a relative and an absolute measure of individual growth. Standard Scale Invariance (SI), respectively

[1] How frequently each initial income levels appears depends on the data considered. See Table B.1 for this information on the Italian sample we use in the empirical illustration.

Addition Invariance (AI), are then introduced to obtain the specific functional form of both measures of individual growth: the proportional, respectively absolute difference between final and initial income.

With regard to the second aspect, our aggregation procedure is similar to that proposed by Demuynck and Van de gaer (2012). The domain we use is the concatenation of all possible income vectors in the first and second period, where in both vectors individuals are ordered according to their position in the initial income vector (from high to low initial income). On this domain, we impose the counterparts of the structural axioms used by Demuynck and Van de gaer (2012). More in particular, we impose Rank Dependent Monotonicity (RDM), which allows to express aggregate growth as a function of the magnitude of growth experienced by each individual and his position in the initial distribution of income. We also impose Relative and Translation Invariance (RI and TI), requiring that the aggregate growth ordering is unaffected when all individual growth numbers are multiplied by the same constant or if the same constant is added to all individual growth numbers, respectively. We then impose Decomposability with respect to Highest Initial Income (D-HII) which requires that aggregate growth only depends on the aggregate growth of the $n-1$ group of initially richest and on the growth of the initially poorest. Further imposing two normative axioms, Population Invariance (PI) and History Dependent Growth Incidence, we obtain our aggregate index of history dependent growth.

Next, we perform an empirical illustration of our theoretical framework. It is aimed at comparing different consecutive two-year growth processes that took place in Italy from 1998 against the growth process 2008-2010. The focus on 2008-2010 stems from the observation that this is the period during which the first wave of the economic crisis took place, hence we investigate the distributional effect of the crisis. By comparing our results with those obtained applying the pro-poor perspective, we also show the applied relevance of the history dependent perspective.

The paper is organized as follows. In Section 2 we introduce the general notation and present our theoretical results. Section 3 applies the framework to the recent economic crisis in Italy (2008-10). Section 4 concludes.

2 The framework

In this Section we characterize two individual measures of growth and the aggregation of these measures into a societal index of history dependent growth. We follow the major branch in the literature on income mobility measurement, in working with a set of observations of individuals' incomes in two periods (see, e.g. Fields and Ok, 1999a). It has the main advantage that we use the income data in the way they are reported in panel data sets; we don't aggregate them into arbitrary quantiles and compute our index directly on the basis of the individual data. We start by defining the notation we will use throughout this paper.

In order to focus on history dependent growth, it is important to keep track of the initial income level of each individual. Hence we focus on vectors $(\mathbf{x}, \mathbf{w}) = (x_1, ..., x_i, ..., x_n, w_1, ..., w_i, ..., w_n)$, where the index i marks the identity of the individual, x_i is individual i's initial (first period) income and w_i is his second period income. Moreover, individuals are ordered on the basis of their initial income, and those that have the same initial income are ordered on the basis of their second period income. Hence we use the domain

$$D^n = \left\{(\mathbf{x}, \mathbf{w}) \in \mathbb{R}^n_{++} \text{ such that } x_1 \geq ... \geq x_i \geq ... \geq x_n \text{ and, if } x_i = x_j \text{ and } w_i > w_j, \text{ then } i < j\right\}.$$

Any vector of observations on first and second period incomes can be reshuffled such that it has a representation in the domain D^n. In that sense the domain is not restrictive. We use it because it allows us to keep track of the rank of each individual in the initial distribution of income.

Our aim is to characterize an index $G^n(\mathbf{x}, \mathbf{w}) : D^n \longrightarrow \mathbb{R}$, where G^n is a non constant function that measures aggregate growth, with special case G^1 measuring the growth experienced by an individual, for which the domain D^1 reduces to $\mathbb{R}^2_{++}$. Let the set $S_i = \{j \in N \text{ such that } x_j = x(i)\}$ contain all individuals that have the i-th highest level of income $x(i)$ and n_i be the cardinality of S_i. We first characterize a measure of individual growth. Next we turn to the aggregation of these individual growth measures.

2.1 Individual growth

We propose a relative and an absolute measure of individual growth. There are good reasons to use either of both measures, a discussion of their pros and cons for a measurement of growth in a history dependent context is outside the scope of this work[2].

Three axioms will be used to characterize both a relative and an absolute measure of individual growth. The first is a standard normalization axiom. It requires that a measure of individual growth should be equal to 0 if the individual does not experience any variation in her level of income.

N (Normalization): For all $x \in \mathbb{R}_{++} : G^1(x, x) = 0$.

The second is a trivial monotonicity axiom: growth is increasing in second period incomes.

M (Monotonicity): For all $x, w, z \in \mathbb{R}_{++} : w > z \Longrightarrow G^1(x, w) > G^1(x, z)$.

The third is an independence condition: for individuals having the same initial level of income, increasing second period incomes changes growth by the same amount, no matter what the original second period level of income is.

IND (Independence): For all $x, w, z \in \mathbb{R}_{++}$ and $\theta > 0$:

$$G^1(x, w + \theta) - G^1(x, w) = G^1(x, z + \theta) - G^1(x, z).$$

As a result of this axiom, changes in second period incomes can only have a differential effect on individuals' growth when these individuals have a different level of initial income. This axiom will be used to cardinalize both the relative and absolute measure in such a way that they become a linear function of second period incomes. Axiom **N** provides further restrictions on the cardinalization.

2.1.1 A measure of relative growth

As is standard, measures of relative growth are scale invariant measures: they are not affected by an equiproportional change in the initial and final level of income.

SI (Scale Invariance): For all $\lambda > 0$ and all $x, w \in \mathbb{R}_{++}$:

[2] For a detailed analysis of this issue in the context of income inequality measurement, see Kolm (1976a,b) and Atkinson and Brandolini (2010).

$$G^1(\lambda x, \lambda w) = G^1(x, w).$$

It is easy to obtain the following Lemma.

Lemma 1: For all $x, v, w, z \in \mathbb{R}_{++}$ the individual growth measure satisfies SI and M if and only if

$$G^1(x, w) > G^1(z, v) \Leftrightarrow \frac{w}{x} > \frac{v}{z}.$$

Lemma 1 says that, if we want to order individual growths in a scale invariant and monotonous way, we have to order them on the basis of their ratios of second to first period incomes. The axioms N and IND are used to cardinalize this ordering, yielding the following.

Proposition 1: A growth measure $G^{1R}(x, w)$ satisfies SI, M, N and IND if and only if there exists $\beta > 0$ such that

$$G^{1R}(x, w) = \beta \frac{(w - x)}{x}.$$

Proposition 1 characterizes a standard measure of individual growth: the proportionate difference between the final and the initial income.

2.1.2 A measure of absolute growth

Measures of absolute growth satisfy addition invariance: the value of the function G^1 does not change if the same amount of income is added to both initial and final income.

AI (Addition Invariance): For all $\theta > 0$ and all $x, w \in \mathbb{R}_{++}$:

$$G^1(x + \theta, w + \theta) = G^1(x, w).$$

It is easy to obtain the following Lemma.

Lemma 2: For all $x, v, w, z \in \mathbb{R}_{++}$ the individual growth measure satisfies AI and M if and only if

$$G^1(x, w) > G^1(z, v) \Leftrightarrow w - x > v - z.$$

Lemma 2 says that if we want to order individual growths in an addition invariant and monotonous way, we have to order them on the basis of their differences between second and first period incomes. The axioms N and IND can be used to cardinalize this ordering. This results in the following.

Proposition 2: A growth measure $G^{1A}(x, w)$ satisfies AI, M, N and IND if and only if there exists $\alpha > 0$ such that

$$G^{1A}(x, w) = \alpha(w - x).$$

Proposition 2 characterizes a standard measure of individual growth: the difference in level between the final and the initial income.

The indices of individual growth obtained in Propositions 1 and 2 have been already introduced in the literature and are widely implemented in empirical works[3]. We provide a unifying framework to derive both indices, using the new independence axiom.

2.2 From individual to aggregate growth

In this Section we characterize a measure of aggregate growth. In order to do so, recall that our framework builds on the assumption that the vector of initial incomes is sorted non-increasingly, and individuals with the same initial income are ordered from high to low final income, while in the vector of final incomes individuals are ordered in the same way -see the definition of the domain D^n. It follows that $G^1(x_i, w_i)$, is the measure of growth of the individual ranked i-th in $\mathbf{x}$.

The structural axioms we use below (RDM, RI, TI and D-HII) have been used in the literature, but on different domains. Bossert (1990) used these axioms on the domain of ordered single period income vectors (individual incomes ordered from high to low) to characterize the generalized Gini social evaluation function. Demuynck and Van de gaer (2012) used them on the domain of ordered mobility vectors (individual mobilities ordered from high to low). We translate the structural axioms used in Demuynck and Van de gaer (2012) to the domain D^n.

We begin with a monotonicity axiom, tailored to study history dependent growth. When comparing two growth processes, the growth process $G^n(\mathbf{x}, \mathbf{w})$ has higher growth than $G^n(\mathbf{v}, \mathbf{z})$ if all individuals that occupy the same position in $\mathbf{x}$ and $\mathbf{v}$ experience higher or equal growth in $G^n(\mathbf{x}, \mathbf{w})$ than in $G^n(\mathbf{v}, \mathbf{z})$, with at least one individual experiencing higher growth in $G^n(\mathbf{x}, \mathbf{w})$ than in $G^n(\mathbf{v}, \mathbf{z})$.

RDM (Rank Dependent Monotonicity): For all $n \in \mathbb{N}$ and all $(\mathbf{x}, \mathbf{w})$ and $(\mathbf{v}, \mathbf{z}) \in D^n$,

$$G^n(\mathbf{x}, \mathbf{w}) > G^n(\mathbf{v}, \mathbf{z}) \text{ if } G^1(x_i, w_i) \geq G^1(v_i, z_i) \text{ for all } i \in \{1, ..., n\},$$

with at least one inequality strictly holding.

This axiom is similar to the Weak Decomposability axiom commonly used in the literature on mobility measurement and allows to express the aggregate measure of growth as a function of each individual's measure of growth. However, it is applied to "history dependent" distributions of income, that is, distributions where the individuals are ordered according to their rank in the initial period. This implies that this axiom makes it possible to express the aggregate measure of growth as a function of the magnitude of each individual growth, while keeping track of their rank in the initial distribution of income.

The next axiom says that comparisons between income growth measures remain invariant when all individual income growth measures are multiplied by the same constant.

RI (Relative Invariance): For all $n \in \mathbb{N}$ and all $(\mathbf{x}, \mathbf{w})$, $(\mathbf{v}, \mathbf{z})$, $(\mathbf{x}, \mathbf{a})$ and $(\mathbf{v}, \mathbf{b}) \in D^n$, if $G^n(\mathbf{x}, \mathbf{w}) = G^n(\mathbf{v}, \mathbf{z})$ and there exists $\lambda > 0$ such that for all $i \in \{1, \ldots, n\}$ we have that $G^1(x_i, a_i) = \lambda G^1(x_i, w_i)$ and $G^1(v_i, b_i) = \lambda G^1(v_i, z_i)$, then $G^n(\mathbf{x}, \mathbf{a}) = G^n(\mathbf{v}, \mathbf{b})$.

[3] There exist evident alternatives, like $\log(w) - \log(x)$ or $(w/x)^r$ with $r > 0$, being the relative growth measures present in the directional income mobility measures of Fields and Ok (1999b), and Schluter and Van de gaer (2011), respectively, or $\exp[c(w - x)]$ with $c > 0$, the absolute growth measure present in another directional mobility measure of Schluter and Van de gaer (2011).

As a result of RDM and RI, the aggregate growth index will be homothetic in individual growths ranked according to initial income levels.

The next axiom says that comparisons between income growth measures remain invariant when the same constant is added to all individual income growth measures.

TI (Translation Invariance): For all $n \in \mathbb{N}$ and all $(\mathbf{x}, \mathbf{w})$, $(\mathbf{v}, \mathbf{z})$, $(\mathbf{x}, \mathbf{a})$ and $(\mathbf{v}, \mathbf{b}) \in D^n$, if $G^n(\mathbf{x}, \mathbf{w}) = G^n(\mathbf{v}, \mathbf{z})$ and there exists $\lambda > 0$ such that for all $i \in \{1, \ldots, n\}$ we have that $G^1(x_i, a_i) = G^1(x_i, w_i) + \lambda$ and $G^1(v_i, b_i) = G^1(v_i, z_i) + \lambda$, then $G^n(\mathbf{x}, \mathbf{a}) = G^n(\mathbf{v}, \mathbf{b})$.

As a result of RDM and TI, the aggregate growth index will be translatable in individual growths ranked according to initial income levels, meaning that all the iso-aggregate growth curves have the same shape, shifted by a constant λ in each direction.

The following axiom says that aggregate growth depends on the aggregate growth measure of the $n-1$ individuals that are ranked first in D^n and the growth measure of the individual that is ranked last in D^n. The latter is the individual with the lowest initial income level. If there is more than one individual with this initial income level, it is that individual with the lowest income growth among those having the lowest initial income. Let $(\mathbf{r}_{-n}, \mathbf{s}_{-n}) = (r_1, \ldots, r_{n-1}, s_1, \ldots, s_{n-1}) \in D^{n-1}$. We can then formulate the axiom as follows.

D-HII (Decomposability with respect to the Highest Initial Income Level): For all $n \in \mathbb{N}$ and all $(\mathbf{x}, \mathbf{w})$ and $(\mathbf{v}, \mathbf{z}) \in D^n$, if $G^{n-1}(\mathbf{x}_{-n}, \mathbf{w}_{-n}) = G^{n-1}(\mathbf{v}_{-n}, \mathbf{z}_{-n})$ and $G^1(x_n, w_n) = G^1(v_n, z_n)$, then $G^n(\mathbf{x}, \mathbf{w}) = G^n(\mathbf{v}, \mathbf{z})$.

Like in the previous papers where a similar axiom has been used (Bossert (1990), Demuynck and Van de gaer (2012)), its purpose is to separate the contribution of the worst-off from the contribution of the others. In our context, the worst-off is the individual with the lowest income growth among the initially poorest; the n-th individual. The combination of the previous axioms results in the following Lemma.

Lemma 3: For all $n \in \mathbb{N}$, an aggregate index of growth G^n satisfies RDM, RI, TI and D-HII if and only if there exist strictly positive coefficients $\gamma_1^n, \gamma_2^n, ..., \gamma_n^n$, such that, for all $(\mathbf{x}, \mathbf{w}) \in D^n$ and corresponding $\mathbf{g} = \left(G^1(x_1, w_1), \ldots, G^1(x_n, w_n)\right) \in \mathbb{R}^n$,

$$G^n(\mathbf{x}, \mathbf{w}) = \sum_{i=1}^{n} \gamma_i^n g_i, \text{ with } \sum_{i=1}^{n} \gamma_i^n = 1.$$

The proof of this Lemma is similar to the proof of Lemma 4 in Demuynck and Van de gaer (2012). The Lemma says that aggregate growth can be written as a weighted average of individuals' growth, with weights dependent on the individual's rank in the domain D^n.

As stated in the introduction, the next axiom is new and crucial in the present analysis. Measures of history dependent growth favor growth redistributions from individuals with a high initial income to individuals with a low initial income and are indifferent between growth redistributions among individuals having the same initial income[4].

[4]We are not claiming that the growth redistributions used in the axiom are feasible. By imposing the axiom, we are only saying that our axiom describes our preference in case the situations described in the axiom occur.

HDGI (History Dependent Growth Incidence): For all $(\mathbf{x},\mathbf{w})$ and $(\mathbf{x},\mathbf{z}) \in D^n$ that are such that for all $i \neq k, l : G^1(x_i, w_i) = G^1(x_i, z_i)$ and there exists a $\Delta > 0$ such that

$$G^1(x_l, z_l) = G^1(x_l, w_l) + \Delta \text{ and } G^1(x_k, z_k) = G^1(x_k, w_k) - \Delta,$$

then

$$\begin{aligned} &\text{(a) if } x_l \leq x_k, \text{ then } G^n(\mathbf{x},\mathbf{w}) \leq G^n(\mathbf{x},\mathbf{z}), \\ &\text{(b) if } x_l = x_k, \text{ then } G^n(\mathbf{x},\mathbf{w}) = G^n(\mathbf{x},\mathbf{z}). \end{aligned}$$

Given the structure in Lemma 3, HDGI imposes straightforward restrictions on the coefficients γ_i^n: when two individuals have the same initial level of income, their γ_i^n must be the same, while when they have a different initial level of income, the one with the lowest level of income gets a weight that is not lower than the weight of the one with the higher initial income. As a result, the index can be written as follows.

Lemma 4: For all $n \in \mathbb{N}$, an aggregate index of growth G^n satisfies RDM, RI, TI, D-HII and HDGI if and only if there exist strictly positive coefficients $\gamma_1^m, \gamma_2^m, ..., \gamma_m^m$, such that, for all $(\mathbf{x},\mathbf{w}) \in D^n$ and corresponding $\mathbf{g} = \left(G^1(x_1, w_1), \ldots, G^1(x_n, w_n)\right) \in \mathbb{R}^m$,

$$\begin{aligned} G^n(\mathbf{x},\mathbf{w}) &= \sum_{i=1}^{m} \gamma_i^m g_i, \text{ with } g_i = \sum_{j \in S_i} G^1(x_j, w_j), \\ \gamma_1^m &\geq \ldots \geq \gamma_m^m \text{ and } \sum_{i=1}^{m} \gamma_i^m n_i = 1 \text{ with } n_i = \# S_i. \end{aligned}$$

Our final axiom is a standard Population Invariance axiom. It states that the measure of aggregate growth is invariant to a k-fold replication of the same vector of initial and final incomes. This property ensures that we can apply this measure to compare growth processes taking place over distributions with different population sizes.

PI (Population Invariance): For all $(\mathbf{x},\mathbf{w}) \in D^n$ and $(\mathbf{y},\mathbf{z}) \in D^{kn}$ that are such that

$$\mathbf{y} = \left(\underbrace{x_1, ..., x_1}_{k\ times}, ..., \underbrace{x_n, ..., x_n}_{k\ times} \right) \quad \text{and} \quad \mathbf{z} = \left(\underbrace{w_1, ..., w_1}_{k\ times}, ..., \underbrace{w_n, ..., w_n}_{k\ times} \right),$$

$$G^n(\mathbf{x},\mathbf{w}) = G^{nk}(\mathbf{y},\mathbf{z}).$$

Following Donaldson and Weymark (1980), population invariance allows us to get a functional form for the weights. Formally,

Proposition 3. For all $n \in \mathbb{N}$, an aggregate index of growth G^n satisfies RDM, RI, TI, D-HII, HDGI and PI if and only if there exists a parameter δ, such that, for all $(\mathbf{x},\mathbf{w}) \in D^n$ and corresponding $\mathbf{g} = \left(G^1(x_1, w_1), \ldots, G^1(x_n, w_n)\right) \in \mathbb{R}^m$,

$$G^n(\mathbf{x},\mathbf{w}) = \sum_{i=1}^{m} \frac{i^\delta - (i-1)^\delta}{\sum_{l=1}^{m}\left(l^\delta - (l-1)^\delta\right) n_l} g_i$$
$$\text{with } g_i = \sum_{j \in S_i} G^1(x_j, w_j),\ n_l = \#S_l \text{ and } \delta \geq 1.$$

Our index of history dependent growth attaches to each individual growth rate a weight that is decreasing in the rank of the individual in the initial income distribution. The parameter δ is a sensitivity parameter: for $\delta = 1$, everybody's growth rate gets the same weight; as δ increases, the relative weight to the initially poorest increases and the weight to the initially richest decreases; as approaches ∞, only the growth rate of the initially poorest matters. If the individual growth measure $G^{1R}(x,w)$, characterized in Proposition 1, is chosen, we obtain an aggregate relative growth measure, if the individual growth measure $G^{1A}(x,w)$, characterized in Proposition 2, is chosen, we obtain an aggregate absolute growth measure. Observe that if all $\#S_l = 1$, the term in the denominator of $G^n(\mathbf{x},\mathbf{w})$ reduces to $1/m^\delta$, which is the standard weight derived for the single-series Gini by Donaldson and Weymark (1980) and $\delta = 2$ gives the standard Gini weights. The denominator of our index in Proposition 3 is more complicated to take into account that different initial income levels can occur with different frequencies in the population.

The value of the index derived in Proposition 3 depends on the value of the sensitivity parameter δ. Abusing notation, we write the index as $G^n(\delta)$ to make this dependency explicit. The index is sensitive to both the distribution of growth among the individuals and the level of growth: doubling all individual growth rates does not affect the distribution of growth, but doubles the value of $G^n(\delta)$. As advocated by Jenkins and Van Kerm (2011), it is interesting to separate the purely distributive effect of the growth process (the " progressivity aspect") from the average growth experienced by the population as a whole. Since $G^n(1)$ equals the average of all individual growths, a natural measure of this progressivity is

$$P^n(\delta) = G^n(\delta) - G^n(1).$$

Actual growth processes differ in both the distribution of growth and the overall level of growth. The progressivity index allows us to compare their purely distributive effects.

Jenkins and Van Kerm (2011) propose a mobility measure in a continuous framework that is closely related to our measure of history dependent growth. Let the distribution of income in the first period, X, be described by the cumulative distribution function $F_X(x)$. The normalized rank in the base-year income distribution corresponding to income level x is $p = F_X(x)$. The counterpart of the individual growth measure is what they call $M(p)$, the "Mobility Profile", and gives the income growth experienced by each percentile of the original income distribution. The mobility measure they propose is

$$\int_0^1 v(1-p)^{v-1} M(p)\, dp \text{ with } v \geq 1.$$

Consider now a discrete set of data that is such that it has a structure that is directly comparable to the structure in Jenkins and Van Kerm: every initial position occurs exactly k times. In that case, let $\widetilde{g}$ be the vector of individual growth rates obtained after ordering the income vectors on the basis of initial incomes from low to high (as in the Mobility Profile). It is then possible to show the following.

Proposition 4. If $(\mathbf{x}, \mathbf{w})$ is such that for all $i = 1, \ldots, n$, $n_i = k$, then the index derived in Proposition 3 can be approximated by the expression

$$\sum_{i=1}^{m} \delta \left(1 - \frac{i}{m}\right)^{\delta-1} \frac{1}{m} \frac{\widetilde{g}_i}{k}.$$

As $\frac{\widetilde{g}_i}{k}$ is average income growth of the individuals in the $i-$th quantile of the distribution of initial income, and $\frac{1}{m}$ the relative frequency with which each initial income level occurs, it follows that the measure of Jenkins and Van Kerm approximates our measure. The advantage of our measure, however, is that it works with discrete data, which is the format of all empirical data; it does not require an arbitrary division of the initial income distribution in quantiles and/ or its computation does not require the non-trivial estimation of the function $M(p)$.

3 The distributional implications of the crisis in Italy

In this Section we implement our theoretical framework in order to investigate changes in the Italian growth process over the last decade. We assess the consequences of the recent economic crisis on the Italian distribution of income from the history dependent perspective. We emphasize the relevance of our indices by showing that their ranking of the growth processes provides information that is difficult to spot with existing tools such as the standard Growth Incidence Curve (GIC) or the more recent Non-anonymous Growth Incidence Curve (na-GIC).

3.1 The data

Our empirical illustration is based on the panel component of the last seven waves of the Bank of Italy "Survey on Household Income and Wealth" (SHIW). The SHIW is a representative sample of the Italian resident population interviewed every two years. In particular, we consider the 1998, 2000, 2002, 2004, 2006, 2008 and 2010 waves.

The unit of observation is the household, defined as all persons sharing the same dwelling. Our measure of living standard is household net disposable income, which includes all household earnings, transfers, pensions, and capital incomes, net of taxes and social security contributions. Household income is expressed in constant prices of 2010 and then adjusted for differences in household size using the OECD equivalence scale (the square root of household size). In line with the literature, for each wave, we drop the bottom and top 1% in the income distribution from the sample to eliminate the effect of the outliers. Table 1 below reports the yearly mean income growth rate and the main features of the government in power in each two year period since 1998[5].

To investigate the distributional impact of the recent economic crisis, we use the growth process 2008-2010 as benchmark since the first wave of the crisis took place in Italy in 2008. We compare all previous two-year period growth processes starting from 1998-2000 with this benchmark. In the main text, for the sake of brevity, we only provide a detailed report for the comparison with the 2004-2006 period, the period immediately preceding the economic crisis. We have chosen this comparison because, apart from the crisis, these adjacent periods are most similar. Moreover, the parties in power were center-rightist in both periods and the Prime Minister (S. Berlusconi) was the same (see Table 1). These periods differ in terms of their mean income growth rate, but our

[5]Information about sample sizes are reported in Table B.1 in Appendix B.

progressivity index eliminates this effect. The comparisons of the other periods with the benchmark yield broadly similar results and are briefly discussed at the end of the next Section; the detailed results are reported in Appendix B for completeness.

Table 1: Mean income growth of yearly equivalized income, political parties and coalitions in power for each period.

Period	Mean income growth rate	Government Political Party	Government Coalition
1998-00	0.0230	Democrati di sinistra	L'Ulivo -UDR (L)
2000-02	0.0159	I Democrati (up2001) / Forza Italia	L'Ulivo (L)/Casa della Liberta (R)
2002-04	0.0181	Forza Italia	Casa delle Liberta (R)
2004-06	0.0207	Forza Italia	Casa delle Liberta (R)
2008-10	-0.0021	Il popolo della liberta	PdL MPA LNP (R)

Note: in the column "coalition", (L) stands for leftist, (R) for rightist.

We use sample weights to compute all estimates[6]. We give each household the sample weight corresponding to the sampling in the first wave of the survey in our analysis (1998). To the households selected into the survey at subsequent waves, we give the sample weight corresponding to the sampling in the wave of their first inclusion into the survey[7]. The standard errors of our estimates are obtained through 1000 bootstrap replications. In particular, in order to take into account the dependence structure of our observations, we use the non-parametric block bootstrap procedure described by Cameron and Trivedi (2010)[8].

3.2 Results

In this Section we show how our measure can be used to investigate the history dependent distributional implications of impressive macroeconomic changes. We compare the growth process 2004-06 against the growth process 2008-10. Before discussing our growth measures we perform a standard pro-poor growth analysis, that is, we compute the Growth Incidence Curves (GICs) for the two growth processes. They plot the growth in mean income levels at the same percentile in the initial and final income distribution as a function of this percentile. They are reported in the left-hand panel of Figure 1.

Some features stand out. First, all growth rates depicted by the GIC for the 2004-06 period are positive, while for the 2008-10 period they are negative for the bottom and top 20%, with large negative values for the bottom part of the distribution. Moreover, there is a dominance relationship between the two processes: the GIC of the first period is always above the GIC of the second. Furthermore, the two growth processes show very different patterns. The first process is characterized by a generally progressive trend with higher growth rates for poorer than for richer quantiles. It is quite progressive up to the percentile 0.60, but becomes almost regressive for the upper part of the distribution. The second process is generally regressive: growth is highly

[6] We use cross-sectional individual sample weights, at time t. As shown by Faiella and Gambacorta (2007) in the case of the SHIW, for the production of longitudinal statistics, there is no unambiguous evidence that the use of longitudinal weights always performs better than cross-sectional weighing in terms of efficiency.

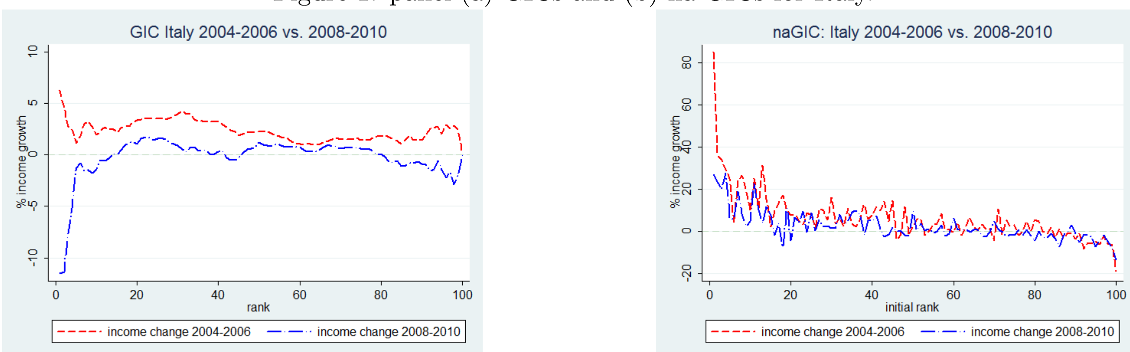
[7] See on this Hildebrand et al. (2012) and Jenkins and Van Kerm (2011).

[8] In this procedure, the bootstrap samples are obtained by implementing the `bsweight` stata routine proposed by Kolenikov (2010), which takes into account the complex survey structure of data.

regressive for the bottom 20% of the distribution, but becomes almost progressive for the rest of the distribution[9].

When we endorse a history dependent perspective, the comparison changes dramatically. A basic tool here is the non-anonymous Growth Incidence Curve (na-GIC) which plots the growth in mean income obtained by households belonging to the same percentile in the initial income distribution as a function of this percentile. The right-hand panel of Figure 1 plots the na-GICs for the periods before and after the financial crisis.

Figure 1: panel (a) GICs and (b) na-GICs for Italy.



The two na-GICs differ markedly from the corresponding GICs, in particular for the 2008-10 process. Both na-GICs are positive up to the 50th percentile, are around zero up to the 85th percentile and become negative for the initially richest percentiles. The two growth episodes also show a similar progressive path. Hence it appears that in both periods the initially poorest gain more from growth than the initially rich. However, we encounter a major difficulty in the comparison of these two growth processes. No dominance can be established since the two curves intersect very often.

The comparison of the GIC and na-GIC shows that adopting a history dependent approach matters. However, the na-GICs are not helpful for ranking growth processes when, as in our case, they overlap frequently. This drawback can be overcome by implementing the measure we derived in the previous Section. The values taken by our measure for each period are reported in Table 2, while the results of comparing the two growth processes are reported in Table 3.

Table 2: History dependent growth indices 2004-06 and 2008-10.

				δ		
		1	2	4	6	8
$G^n_{04/06}(\delta)$	Relative	0.0596	0.1067	0.1582	0.1932	0.2217
	Absolute	420.6	1025.7	1310.9	1449.4	1550.2
$G^n_{08/10}(\delta)$	Relative	0.0239	0.0516	0.0793	0.0970	0.1108
	Absolute	-43.1	426.5	625.5	703.6	754.2

Note: authors' calculations, based on the Italian "Survey on Household Income and Wealth".

[9]The results for the 2008-10 period is in line with what has been found by Jenkins et al. (2011) through microsimulation estimations, who show that the crisis has acted by increasing the aggregate index of inequality.

Table 3: Test of the hypothesis $G^n_{04/06}(\delta) - G^n_{08/10}(\delta) > 0$.

	δ				
	1	2	4	6	8
Relative	0.0357***	0.0554***	0.0789***	0.0963***	0.1109***
Absolute	463.73***	599.43***	685.40***	745.74***	795.96***

Note: authors' calculations, based on the Italian "Survey on Household Income and Wealth". *** means statistically significant at 99 %, ** statistically significant at 95 % and * statistically significant at 90 %.

For all values of the sensitivity parameter δ, our index measures more history dependent growth during the 2004-06 process than during the 2008-10 process. This holds both when a relative and an absolute index is used. The difference is always statistically significant. Hence it can be inferred that the 2004-06 growth episode is better than the 2008-10 according to our measure of history dependent (relative and absolute) growth. As the 2004-06 na-GIC curve dominates the 2008-10 curve for the bottom 10% of the initial distribution, while the two curves overlap for the top 10%, this information is, in part, also captured by the na-GICs. However, our measure appears to be more powerful since it generates a clear ranking of the two growth processes.

Looking at the value taken by the indices, it is striking that the divergence in the performance of the two processes increases with the value of δ. For example, when $\delta = 1$, the value of the relative index for the first period is about 0.0357 points higher than the same value for the second period growth index. When $\delta = 8$ there is a divergence of about 0.1109 between the two history dependent relative indices. Recall that the higher is δ the higher is the contribution of the growth experienced by the initially poorest individuals in the evaluation of overall growth. This confirms that the consequences of the crisis are actually hurting the initially poorest individuals most.

Focusing on the value of the index when $\delta = 1$, that is when all individual growth experiences get the same weight such that history dependency is not taken into account, the difference between the indices of the two processes considered is still very great. This might imply that the result when history dependency is taken into account ($\delta > 1$), is mostly due to the overall level of growth. In order to investigate this issue, we adopt the solution given at the end of Section 2.3; that is, we compare the progressivity indices to compare the pure distributional effect of both processes.

The results, reported in Table 4 show that, even when the focus is on the pure distributional effect of growth, the 2004-06 growth process remains more desirable from a history dependent perspective than the 2008-10 growth process (although, the result is not significant when $\delta = 2$, and it is only significant at 90% for the absolute index). Thus, both the overall extent of growth and the pure distributional effect play a role in ranking the growth process of 2004-06 above the growth process 2008-2010 from a history dependent perspective.

Table 4: Test of the hypothesis $P^n_{04/06}(\delta) > P^n_{08/10}(\delta)$.

	δ			
	2	4	6	8
Relative	TRUE	TRUE**	TRUE**	TRUE**
Absolute	TRUE	TRUE*	TRUE*	TRUE*

Note: authors' calculations, based on the Italian "Survey on Household Income and Wealth". *** means statistically significant at 99 %, ** statistically significant at 95 % and * statistically significant at 90 %.

In order to put the distributional implications of the crisis into further perspective, it is interesting to describe briefly the results of the other comparisons. In fact, notice that according to our family of indices also the 1998-00 episode outperforms the 2008-10 episode. The difference in the value of their corresponding indices of history dependent growth is also impressive. Moreover, the 1998-00 process also performs better than the 2008-10 process when only distributional aspects are taken into consideration; this turns out to be statistically significant for all values of δ (see Tables B.5, B.6 and B.7 in Appendix B.2). Similar information can be grasped from the comparison 2008-10 versus 2000-02 and 2002-04. The period of the crisis performs always worst than the growth episodes in 2000-02 and 2002-04. The dominance of the 2000-02 and the 2002-04 over the 2008-10 is statistically significant for every value of δ with both a relative and an absolute measure of individual growth. Most importantly, the sign of the dominance remains the same and it is statistically significant, although there are some exceptions when the pure distributive aspect of growth is considered[10]. However, it is interesting to notice that, among all the growth episodes analyzed, the 1998-00 is the most pro-poor, which is also the only period with a leftist government in power. This may be due to progressive tax-benefit system reforms. The most important reforms were a reduction in the number of the personal income tax brackets (from 7 to 5); a reduction of the maximum marginal tax rate (from 51% to 45.5%); an increase in the minimum marginal tax rate (from 10 to 18.5%); an increase in the amount of the tax allowance for households with children; the introduction of a tax allowance for households with at least one child younger than three; the introduction of a tax allowance for pensioners, which is increasing for pensioners older than 75; the introduction of additional family benefits, such as a maternity benefit and a benefit for households with at least three minors[11]

A last observation is in order here. Taken together, the results of these comparisons emphasize that the crisis is hurting the Italian population. The crisis period not only reduced the level of the growth rates, but income growth has also been redistributed away from the poor.

Among the causes of this result, a relevant role is played by the liberalization of the Italian labor market, in 1997-98 through the so-called “Treu law” (law 196/1997). The Treu-law regulated some forms of flexible work such as apprenticeship and internship. Most importantly it legalized the supply of temporary workers by the Temporary Work Agencies (TWA), which were forbidden until then (due to a law introduced in 1960). The TWA’s rapidly expanded during the following decade. On the one hand, this expansion made entering the labor market easier, especially for unskilled workers, which are likely to belong to the group of the initially poorest individuals in our analysis. Hence the TWA operations led to a sudden increase in the incomes of the initially poorest. On the other hand, these structural changes played a bad role by aggravating the severe distributional implications of the crisis. In fact, they increased wage flexibility and job turnover giving more job opportunities during periods of consistent positive growth (from 1998 to 2006) but during slowdown and recessions (2008-10), workers with atypical job contracts have become more likely to be fired and cannot benefit from social protection[12].

At the same time, these results show, to some extent, the ineffectiveness of the policy interventions carried out by the government during the two-year period 2008-10 to alleviate the impact of the crisis on the weakest segments of the population. These interventions include the tax exemption for overtime, the introduction of a family benefit[13] and the social card[14]. As previous

[10]For the comparisons 2002-04 versus 2008-10 the difference in progressivity is not significant when an absolute measure with $\delta = 2$ is chosen. For the comparison 2000-02 it is not significant when an absolute measure with $\delta = 2, 4$ or 6 is considered.

[11]For a discussion of the progressive effects of these reforms, see, Baldini et al (2002 and 2006).

[12]See Jappelli and Pistaferi (2009) for a detailed analysis.

[13]*una tantum*: a one-off monetary benefit for low-income households.

[14]A voucher for general expenditures for households with elderly people (over 65) or with at least one child younger

publications have shown [15] , the ineffectiveness of these actions could have been caused by the specific target of the population they were designed for: they include some needy households, but excluded other even more, needy ones. The tax exemption for overtime, for instance, benefits only employed individuals. Similarly, eligibility for the social card was based on income, wealth and age requirements, implying that most of the households benefiting from it were households with elderly people, whereas other kinds of households, such as single parent families or families with many children were excluded from this benefit.

4 Conclusions

The size of the recent economic crisis begs the question of the distributional impact of the crisis. More in particular, we want to know whether the crisis is affecting more the initially poor or the initially rich. This is a history dependent approach since it takes into account individuals' initial economic conditions.

Endorsing this perspective, we have provide a characterization of a synthetic index of history dependent growth. The crucial steps in the characterization are the definition of the domain, which allows to keep track of individuals' position in the initial income distribution and the history dependent growth axiom, which prefers redistributions of growth to the initially poorest and is indifferent between growth redistributions between individuals having the same initial income. The resulting index of history dependent growth is expressed as a weighted average of the growth experienced by each single individual, with weights that decrease with the rank of these individuals in the initial distribution of income. Our index turns out to be closely related to the mobility measure of Jenkins and Van Kerm (2011), but is easier to compute, and, like their index, it is additively decomposable into a pure distributive effect and the mean growth rate.

We have shown the applicability of our framework with an empirical application in which we describe effects of the economic crisis on the Italian population. We find that the Growth Incidence Curves and the non-anonymous Growth Incidence Curves have very different shapes, such that the history dependent and the pro-poor perspective can lead to different conclusions when comparing growth processes. Moreover, na-GICs cross frequently, making it impossible to obtain clear conclusions about the ranking of growth processes. Our measure allows us to obtain also in such situations a clear ranking. Concerning the impact of the economic crisis on Italian households, we find that the growth process during the crisis is worse from a history dependent perspective than any of the preceding growth processes, even when we correct for the differences in mean growth and only consider the way growth is distributive. This is clear evidence that the economic crisis is hitting the initially poor disproportionately.

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A Theoretical Appendix

A.1 Proof of Lemma 1

Proof. The proof is simple: first apply SI, define the function $\hat{G}^1(x) = G^1(1, x)$ and then apply M, to get

$$G^1(x, w) > G^1(z, v) \Leftrightarrow G^1\left(1, \frac{w}{x}\right) > G^1\left(1, \frac{z}{v}\right) \Leftrightarrow \hat{G}^1\left(\frac{w}{x}\right) > \hat{G}^1\left(\frac{z}{v}\right) \Leftrightarrow \frac{w}{x} > \frac{v}{z}.$$

as stated in the Lemma. □

A.2 Proof of Proposition 1

Proof. From IND and the definition of the function $\hat{G}^1(x)$, we get

$$\hat{G}^1\left(\frac{w+\theta}{x}\right) - \hat{G}^1\left(\frac{w}{x}\right) = \hat{G}^1\left(\frac{z+\theta}{x}\right) - \hat{G}^1\left(\frac{z}{x}\right).$$

With a trivial redefinition of variables, this becomes that for all a, b and $c \in \mathbb{R}_{++}$,

$$\hat{G}^1(a+c) - \hat{G}^1(a) = \hat{G}^1(b+c) - \hat{G}^1(b),$$

which implies that the function $\hat{G}^1$ must be linear: $\hat{G}^1(x) = \alpha + \beta x$, such that

$$G^1(x, w) = \hat{G}^1\left(\frac{w}{x}\right) = \alpha + \beta\frac{w}{x}.$$

Due to N, we get $\alpha = -\beta$, and from M, $\beta > 0$ from which the result follows. □

A.3 Proof of Lemma 2

Proof. In order to prove the Lemma, we distinguish 4 cases.

(i) If $w > x$ and $v > z$, apply AI, define the function $\tilde{G}^1(x) = G^1(0, x)$ and then apply M, to get

$$G^1(x, w) > G^1(z, v) \Leftrightarrow G^1(0, w-x) > G^1(0, v-z) \Leftrightarrow \tilde{G}^1(w-x) > \tilde{G}^1(v-z) \Leftrightarrow w - x > v - z.$$

(ii) If $w < x$ and $v < z$, apply AI, define the function $\tilde{\tilde{G}}^1(x) = G^1(x, 0)$ and then apply M, to get

$$G^1(x, w) > G^1(z, v) \Leftrightarrow G^1(x-w, 0) > G^1(z-v, 0) \Leftrightarrow \tilde{\tilde{G}}^1(x-w) > \tilde{\tilde{G}}^1(z-v) \Leftrightarrow w - x > v - z.$$

(iii) If $w > x$ and $v < z$, then $G^1(x, w) > G^1(z, v)$ for every growth measure satisfying AI and M. This follows from M, AI and M again, which yields

$$G^1(x, w) > G^1(x, x) = G^1(z, z) > G^1(z, v).$$

(iv) If $w < x$ and $v > z$, then $G^1(x, w) > G^1(z, v)$ can never hold for any growth measure satisfying AI and M. This follows from M, AI and M again, which yields

$$G^1(x, w) < G^1(x, x) = G^1(z, z) < G^1(z, v).$$

Cases (ii) and (iv) hold automatically for every growth ordering satisfying AI and M, and therefore these cases have no bite. The lemma follows since it holds for all $x, v, w, z \in \mathbb{R}_{++}$. $\square$

A.4 Proof of Proposition 2

Proof. We only prove the case where ($w > x$ and $z > x$). From IND and the definition of the function $\tilde{G}^1(x)$, we get

$$\tilde{G}^1(w + \theta - x) - \tilde{G}^1(w - x) = \tilde{G}^1(z + \theta - x) - \tilde{G}^1(z - x).$$

With a trivial redefinition of variables, this becomes that for all a, b and $c \in \mathbb{R}_{++}$,

$$\tilde{G}^1(a + b) - \tilde{G}^1(a) = \tilde{G}^1(b + c) - \tilde{G}^1(b),$$

which implies that the function $\tilde{G}^1$ must be linear: $\tilde{G}^1(x) = \alpha + \beta x$, such that

$$G^1(x, w) = \tilde{G}(w - x) = \alpha + \beta(w - x).$$

Due to N, we get $\alpha = 0$, and from M, $\beta > 0$ from which the result follows.

The case where θ is such that both $x > w + \theta$ and $x > z + \theta$ can be developed similarly to show that the function $\tilde{G}^1$ is equal to G^{1A}. $\square$

A.5 Proof of Lemma 3

Proof. See Demuynck and Van de gaer (2012). $\square$

A.6 Proof of Proposition 3

Proof. We show that population invariance gives leads to the same functional equation in our context as in Donaldson and Weymark (1980). The result then follows from their Lemma 1 and Theorem 2. Rescale the index in Lemma 4 to get

$$G^n(\mathbf{x}, \mathbf{w}) = \sum_{i=1}^{m} \frac{\gamma_i^m}{\sum_{l=1}^{m} \gamma_l^m n_l} g_i.$$

Define the function $f : \{0, I\} \to \mathbb{R}$ by

$$f(0) = 0, f(1) = 1, \text{ and for } k \neq 0, 1 \text{ we define } f(k) = \sum_{i=1}^{k} \gamma_i^m,$$

and define

$$D^m = \sum_{l=1}^{m} [f(l) - f(l-1)]\, n_l.$$

Now take a $q-$ fold replication of the income vectors, such that by population invariance $G^{qn} = G^n$, which means that

$$\frac{1}{D^{qm}} \sum_{i=1}^{qm} \gamma_i^m g_i = \frac{1}{D^m} \sum_{i=1}^{m} \gamma_i^m g_i. \tag{1}$$

Consider the term behind the summation sign at the LHS of the equality sign. It equals

$$(\gamma_1 + \ldots + \gamma_q)\, g_1 + (\gamma_{q+1} + \ldots + \gamma_{2q})\, g_2 + \ldots \left(\gamma_{(m-1)q+1} + \ldots + \gamma_{mq}\right) g_m$$

Since $\gamma_i = f(i) - f(i-1)$, it is easy to see that

$$\gamma_{lq+1} + \ldots + \gamma_{lq} = f(lq) - f((l-1)\, q)\,,$$

such that

$$\frac{1}{D^{qm}} \sum_{i=1}^{qm} \gamma_i^m g_i = \frac{1}{D^{qm}} \sum_{i=1}^{m} [f(qi) - f(q(i-1))]\, g_i.$$

Moreover,

$$D^{qm} = (\gamma_1 + \ldots + \gamma_q)\, n_1 + (\gamma_{q+1} + \ldots + \gamma_{2q})\, n_2 + \ldots \left(\gamma_{(m-1)q+1} + \ldots + \gamma_{mq}\right) n_m,$$

which in view of the previous result means that

$$D^{qm} = \sum_{i=1}^{m} [f(qi) - f(q(i-1))]\, n_i,$$

and by (1), population invariance requires that

$$\sum_{i=1}^{m} \frac{f(qi) - f(q(i-1))}{\sum_{l=1}^{m} [f(ql) - f(q(l-1))]\, n_l} g_i = \sum_{i=1}^{m} \frac{f(i) - f(i-1)}{\sum_{l=1}^{m} [f(l) - f(l-1)]\, n_l} g_i. \tag{2}$$

From the proof in Donaldson and Weymark (1980), we know when all $n_l = 1$, the solution that satisfies the population principle must be such that

$$f(ql) = f(q)\, f(l) \text{ for all } q, n \in I.$$

It is easy to verify that this solution also satisfies (2). The solution to this functional equation (see Donaldson and Weymark (1980), theorem 2) is $f(l) = l^\delta$, with δ an arbitrary constant, such that $\gamma_l^m = l^\delta - (l-1)^\delta$. Hence $\gamma_l^m \leq \gamma_{l+1}^m$ if and only if l^δ is convex, which requires $\delta \geq 1$. $\square$

A.7 Proof of Proposition 4

Proof. Take the mobility measure derived in Proposition 3, with $n_l = k$.

$$G^n(\mathbf{x},\mathbf{w}) = \sum_{i=1}^{m} \frac{i^\delta - (i-1)^\delta}{m^\delta} \frac{g_i}{k}.$$

Define the vector $\widetilde{g}$ such that, for each $i = 1, \ldots, m$, $\widetilde{g}_i = g_{m+1-i}$: observations are now ordered from the lowest initial income to the highest. The index can then be rewritten as

$$G^n(\mathbf{x},\mathbf{w}) = \sum_{i=1}^{m} \frac{(m+1-i)^\delta - (m-i)^\delta}{m^\delta} \frac{\widetilde{g}_i}{k} = \sum_{i=1}^{m} \left[\left(1 - \frac{i}{m} + \frac{1}{m}\right)^\delta - \left(1 - \frac{i}{m}\right)^\delta\right] \frac{\widetilde{g}_i}{k}.$$

Using the first order expansion,

$$\left(1 - \frac{i}{m} + \frac{1}{m}\right)^\delta \approx \left(1 - \frac{i}{m}\right)^\delta + \delta\left(1 - \frac{i}{m}\right)^{\delta-1} \frac{1}{m},$$

we obtain

$$G^n(\mathbf{x},\mathbf{w}) \approx \sum_{i=1}^{m} \delta\left(1 - \frac{i}{m}\right)^{\delta-1} \frac{1}{m} \frac{\widetilde{g}_i}{k}.$$

□

B Empirical Appendix

B.1 Descriptive statistics

Table B.1: Sample description and the initial income level frequencies for the periods 1998-00, 2000-02, 2002-04, 2004-06 and 2008-10.

	1998-00	2000-02	2002-04	2004-06	2008-10
Sample size	3740	3504	3491	3848	4510
Total number of initial income levels	3661	3400	3393	3722	4385
Individuals per initial income level	Number of initial income levels				
1	3596	3316	3317	3633	4298
2	53	70	70	63	63
3	9	10	13	19	13
4	2	4	3	4	6
5	0	1	1	2	5
6	0	0	0	1	0
7	1	0	0	0	0

Note: The top part of the table reports the total nmber of observations and the number of distinct initial income levels for each growth process considered in this paper. The bottom part reports the frequency of each initial income level for each growth process.

B.2 Comparison: 1998-2000 versus 2008-2010

Figure B.1: panel (a) GICs and (b) na-GICs for Italy.

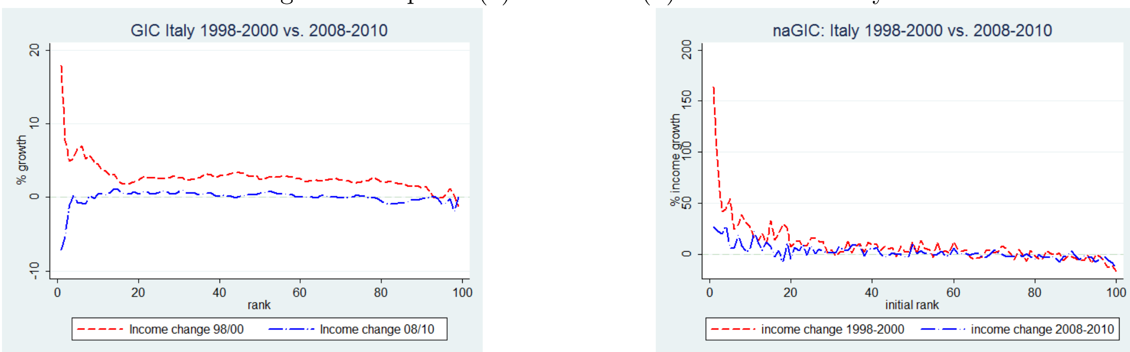


Table B.2: History dependent growth indices 1998-00 and 2008-10.

		δ				
		1	2	4	6	8
$G^n_{98/00}(\delta)$	Relative	0.0965	0.1725	0.2689	0.3389	0.3957
	Absolute	432.2	1136.4	1509.9	1688.3	1805.0
$G^n_{08/10}(\delta)$	Relative	0.0239	0.0516	0.0793	0.0970	0.1108
	Absolute	-43.1	426.5	625.5	703.6	754.2

Note: authors' calculations, based on the Italian "Survey on Household Income and Wealth".

Table B.3: Test of the hypothesis $G^n_{98/00}(\delta) - G^n_{08/10}(\delta) > 0$.

	δ				
	1	2	4	6	8
Relative	0.0727***	0.1210***	0.1896***	0.2419***	0.2849***
Absolute	475.3***	710.0***	884.4***	984.6***	1050.8***

Note: authors' calculations, based on the Italian "Survey on Household Income and Wealth". *** (**) [*] means statistically significant at 99 (95) [90] %.

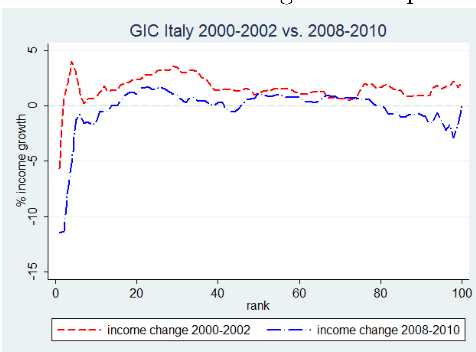
Table B.4: Test of the hypothesis $P^n_{98/00}(\delta) > P^n_{08/10}(\delta)$.

	δ			
	2	4	6	8
Relative	TRUE***	TRUE***	TRUE***	TRUE***
Absolute	TRUE**	TRUE***	TRUE***	TRUE***

Note: authors' calculations, based on the Italian "Survey on Household Income and Wealth". *** (**) [*] means statistically significant at 99 (95) [90] %.

B.3 Comparison: 2000-2002 versus 2008-2010

Figure B.2: panel (a) GICs and (b) na-GICs for Italy.



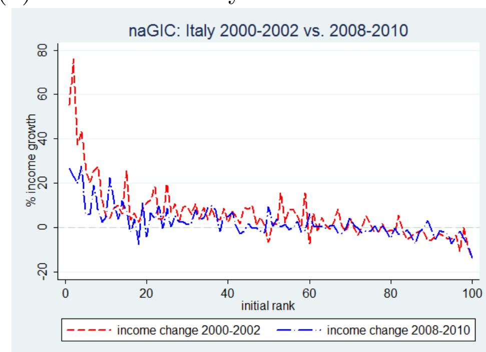


Table B.5: History dependent growth indices 2000-02 and 2008-10.

		δ				
		1	2	4	6	8
$G^n_{00/02}(\delta)$	Relative	0.0604	0.1099	0.1658	0.2055	0.2387
	Absolute	309.8	893.5	1160.1	1281.6	1373.9
$G^n_{08/10}(\delta)$	Relative	0.0239	0.0516	0.0793	0.0970	0.1108
	Absolute	-43.1	426.5	625.5	703.6	754.2

Note: authors' calculations, based on the Italian "Survey on Household Income and Wealth".

Table B.6: Test of the hypothesis $G^n_{00/02}(\delta) - G^n_{08/10}(\delta) > 0$.

	δ				
	1	2	4	6	8
Relative	0.0365***	0.0583***	0.0865***	0.1085***	0.1279***
Absolute	352.9***	467.1***	534.6***	578.0***	619.7***

Note: authors' calculations, based on the Italian "Survey on Household Income and Wealth". *** (**) [*] means statistically significant at 99 (95) [90] %.

Table B.7: Test of the hypothesis $P^n_{00/02}(\delta) > P^n_{08/10}(\delta)$.

	δ			
	2	4	6	8
Relative	TRUE*	TRUE**	TRUE***	TRUE***
Absolute	TRUE	TRUE	TRUE	TRUE*

Note: authors' calculations, based on the Italian "Survey on Household Income and Wealth". *** (**) [*] means statistically significant at 99 (95) [90] %.

B.4 Comparison: 2002-2004 versus 2008-2010

Figure B.3: panel (a) GICs and (b) na-GICs for Italy.

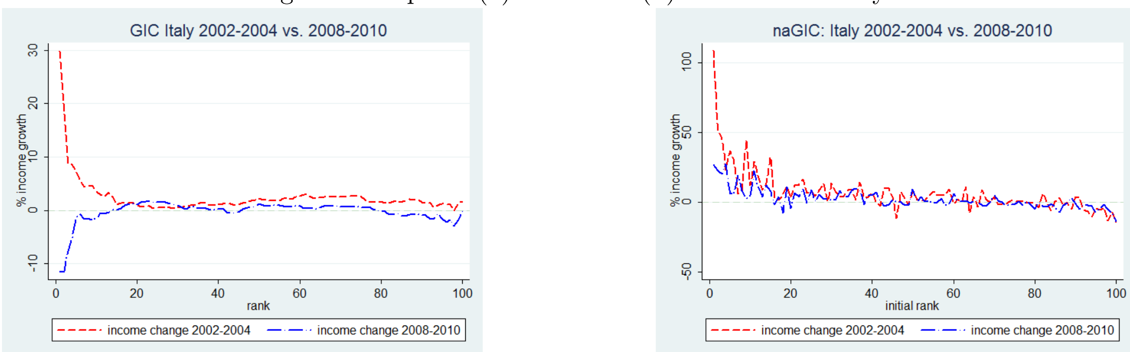


Table B.8: History dependent growth indices 2002-04 and 2008-10.

		δ				
		1	2	4	6	8
$G^n_{02/04}(\delta)$	Relative	0.0656	0.1227	0.1963	0.2518	0.2975
	Absolute	366.0	996.6	1341.8	1519.9	1641.8
$G^n_{08/10}(\delta)$	Relative	0.0239	0.0516	0.0793	0.0970	0.1108
	Absolute	-43.1	426.5	625.5	703.6	754.2

Note: authors' calculations, based on the Italian "Survey on Household Income and Wealth".

Table B.9: Test of the hypothesis $G^n_{02/04}(\delta) - G^n_{08/10}(\delta) > 0$.

	δ				
	1	2	4	6	8
Relative	0.0418***	0.0712***	0.1171***	0.1548***	0.1867***
Absolute	409.1***	570.1***	716.3***	816.2***	887.6***

Note: authors' calculations, based on the Italian "Survey on Household Income and Wealth". *** (**) [*] means statistically significant at 99 (95) [90] %.

Table B.10: Test of the hypothesis $P^n_{02/04}(\delta) > P^n_{08/10}(\delta)$.

	δ			
	2	4	6	8
Relative	TRUE*	TRUE**	TRUE***	TRUE***
Absolute	TRUE	TRUE**	TRUE***	TRUE***

Note: authors' calculations, based on the Italian "Survey on Household Income and Wealth". *** (**) [*] means statistically significant at 99 (95) [90] %.