

Equality of Opportunity and Efficiency in Tertiary Education: a Data-Driven Perspective

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Abstract

This paper examines the evolution of inequality of opportunity (EIOp) and the efficiency of public spending in tertiary education across 31 countries using European Social Survey data for 2010, 2018, and 2023. EIOp is estimated using the conditional inference forests (CIFs) algorithm, with model accuracy assessed using the area under the receiver operating characteristic curve (AUC-ROC). A two-stage Data Envelopment Analysis (DEA) assesses the efficiency of public spending in tertiary education. The findings indicate a general post-COVID-19 decline in EIOp, while cross-country disparities persist. Moreover, for the overall country sample, it would be theoretically possible to reduce the Dissimilarity index of inequality of opportunity, adjusted for the share of tertiary graduates, by 31% in 2010 and 22% in 2018 with current resource levels. A Tobit regression explores exogenous factors associated with efficiency scores. The results are robust to a set of sensitivity analyses and offer a benchmark for policymakers seeking to enhance both fairness and efficiency in higher education.

Keywords: inequality of opportunity; tertiary education; machine learning; two-stage DEA; public spending

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1 Introduction

Equality of opportunity is a conception of social justice that is usually assessed on equally desirable outcomes for all individuals in a society, such as income, consumption, or health. However, the role of the educational dimension is also a growing concern, as not everyone has the opportunity to reach the desired level of education. Higher education is fundamental for developing advanced skills and is generally associated with improved employment opportunities and higher earnings OECD (2024). Many barriers – such as social background – still play a significant role in determining who can pursue post-secondary studies. This challenge is not just a matter of personal success, but it can also limit social progress, growth, and innovation as a whole. By reducing the influence of socioeconomic background on an individual's ability to succeed, higher education can break the intergenerational transmission of disadvantage Palmisano et al. (2022).

According to the concept of equality of opportunity, individual achievements are influenced by both circumstances beyond one's control and individual effort, reflecting personal responsibility Roemer (1998). Applying these principles allows us to quantify to what extent inequalities in higher education can be attributed to factors beyond an individual's control.

Pursuing greater equality of opportunity is not only intrinsically desirable, but could enhance economic efficiency since disparities between socioeconomic groups can lead to inefficiencies. As a result, this concept is seen as essential for grasping the relationship between fairness and economic efficiency Ferreira and Walton (2005). Roemer (2006) raises the question of whether "efficiency" refers to "technical" efficiency, typically described in productivity analysis, or "Pareto" efficiency. In what follows, the term efficiency is understood as technical efficiency, meaning the ability to convert inputs into outputs using available resources.

This paper contributes to the literature in two main ways. First, it leverages a machine learning algorithm, specifically Conditional Inference Forests (CIFs)(Breiman (2001);Hothorn et al. (2006)), to examine the extent of inequality of opportunity in tertiary education (EIOp) in 31 countries using data from the European Social Survey (ESS) for 2010, 2018, and 2023. This application extends machine learning inference on inequality of opportunity to the higher education sector while also adapting the analysis to a binary response variable. Second, the paper relates these estimates to a two-stage Data Envelopment Analysis (DEA) to assess the technical efficiency of public spending in maximizing both equity and tertiary education attainment.

To estimate the inequality in the counterfactual distribution as accurately as possible, the model specification that demonstrates the best out-of-sample predictive performance is selected. Specifically, for each country and year, the model that maximizes the Area Under the ROC Curve (AUC) is chosen, using 5-fold cross-validation to determine the optimal value of the α parameter in the CIFs algorithm. As a result, country rankings in terms of EIOp might depend on the chosen model specifications Brunori et al. (2019b). The results reveal that Conditional Inference Forests with the alpha parameter endogenously tuned performs better than the standard probit model and Conditional Inference Tree (CIT) algorithm in almost all cases.

Educational outcomes, though significant, do not represent the sole criterion for evaluating education systems. Schools serve functions beyond the development of academic skills, including fostering equity and equal opportunities Sutherland (2023). In addition to exploring optimal strategies for enhancing the efficiency of spending in the sector through appropriate funding mechanisms, the academic and institutional debate has also examined how various funding models impact equity and performance Agasisti (2023).

This efficiency analysis departs from the conventional approach of maximizing test scores or graduation rates as output. Instead, it is considered as the social maximandum an inverted dissimilarity index of EIOp, weighted by the share of tertiary graduates and expressed as a function of public spending. In other words, it is assumed that national governments want to maximize equality of opportunity and increase the attainment of tertiary education¹.

The results reveal substantial differences in the EIOp between northern and southern European countries. While post-pandemic trends indicate a reduction in EIOp for almost all countries, these regional disparities persist. In the subset of countries included in the DEA analysis, technical efficiency has improved by approximately 10% between 2010 and 2018. However, a gap of 22% remains from full efficiency, suggesting that, in theory, improving the output indicator with current resource levels could be possible. Norway and Belgium are the countries that lie on the efficient frontier in both periods.

The robustness of the results is confirmed through a range of sensitivity analyses aimed at examining the sensitivity of EIOp rankings to sample size, assessing the overall accuracy of the estimates, and implementing the DEA analysis for all countries in each period, rather than only to those with common data availability.

The paper is organized as follows. Section 2 reviews the relevant background literature; section 3 outlines the theory beyond inequality of opportunity; section 4 describes the methodology to estimate EIOp; section 5 describes two-stage DEA for efficiency estimation; section 6 describes the data; section 7 presents the results with some robustness checks; section 8 concludes and discusses some policy implications.

 $^{^{1}}$ Of course, what may be efficient from the viewpoint of equalizing opportunities, may be inefficient from the viewpoint of maximizing GNP per capita Roemer (1998); conversely, it is hardly conceivable that a government's objective would be confined exclusively to the promotion of equity, without also pursuing measurable improvements in educational attainment.

2 Background

This paper contributes to two strands of the existing literature. The first investigates the role of individual background in shaping inequality of opportunity in tertiary education. The second explores technical efficiency in the tertiary education sector, with a specific focus on the role of public expenditure.

Inequality of opportunity in higher education has received relatively less attention in the equality of opportunity literature, partly due to the lack of sufficient data. However, some notable exceptions exist, such as the studies by Peragine and Serlenga (2008) and Brunori et al. (2012), which examine EIOp in Italy. Peragine and Serlenga (2008) use data on final graduation scores and the earnings of individuals with a university degree to highlight the significant influence of family background on students' academic performance and their transition into the labor market. Brunori et al. (2012) focus on equality of opportunity in accessing higher education and provide evidence of a decrease in educational inequality opportunities, particularly between 1998 and 2001. In a more recent study, Jaoul-Grammare and Magdalou (2013) assess the French higher education system by comparing data from 1992 and 2004, finding persistent inequality of opportunity in both years, with a growing trend over the period analyzed. Palmisano et al. (2022) provide comparable lower-bound parametric estimates of EIOp across 31 European countries using two waves (2005 and 2011) of the EU-SILC dataset that include information on family background. The results reveal significant macro-regional differences, aligning with the EIOp estimates obtained in this study.

The second body of literature addressed by this paper concerns technical efficiency. Most research on efficiency in tertiary education sector focuses on individual institutions, such as universities (see Dipierro and De Witte (2024), among others), rather than evaluating the overall efficiency at country level. However, some studies have examined the efficiency of tertiary education at the systemic level. Aubyn et al. (2009) analyze the efficiency of public tertiary education systems across EU countries, Japan, and the United States using semiparametric methods and stochastic frontier analysis. Their findings identify a core group of efficient countries and highlight the importance of secondary education quality, outputbased funding mechanisms, and institutional autonomy in improving efficiency. Agasisti (2011) conducts a cross-country efficiency analysis of European higher education systems using DEA. The study examines public funding mechanisms and finds that Switzerland and the United Kingdom are among the most efficient systems. Kosor et al. (2019) evaluate technical efficiency of public spending on higher education in the EU-28 using DEA. Their analysis identifies both highly efficient systems and those with significant potential for improvement, emphasizing the role of institutional competition in enhancing efficiency. Public sector efficiency analyses typically focus on total government spending or on specific expenditure items. Inequality indices are not commonly employed as outputs in DEA, and studies that incorporate them primarily focus on economic growth or social spending (see Afonso et al. (2010); Lábaj et al. (2014), among others). Empirical evidence indicates a negative association between public spending and inequality of opportunity (IOp) in different contexts, suggesting that targeted public investments may contribute to reducing opportunity gaps (Marrero and Rodríguez (2012); Ferreira and Gignoux (2014); Checchi et al. (2016); Palmisano et al. (2022)). However, this literature does not explain any causal relation and it is evident that the mechanisms through which public spending influences inequality of opportunity are not exclusive. Other areas of public expenditure and exogenous factors may also impact efficiency. Therefore, these potential confounding factors will be evaluated in the second stage of the analysis.

3 Theoretical foundations of inequality of opportunity

Initial approaches in economic theory to conceptualize equality of opportunity aimed to address directly the concept of opportunities (see Kranich (1996); Herrero et al. (1998); Ok (1997); Ok and Kranich (1998); Savaglio and Vannucci (2007); Weymark (2003), among others). The effectiveness of these methods depends largely on the chosen criteria to assess the spectrum of opportunities available to an individual, which are generally difficult to observe. The second group of methods for addressing inequality of opportunity takes an indirect approach and focuses on the effects of how opportunities are distributed (Roemer (1993); Van De Gaer (1995); Fleurbaey (1994); Bossert (1995)). A key contribution of this indirect approach is its development of two core ethical principles fundamental to opportunity egalitarianism (Peragine and Ferreira (2015)). The first principle, known as the principle of compensation, argues that inequalities arising from circumstances outside an individual's control are ethically unjustifiable (Fleurbaey (1995)). The second principle, called the reward principle, focuses on linking outcomes with individual effort, suggesting that inequalities due to different levels of effort are ethically acceptable, without compensating for differences in outcomes that result from such efforts. Although these two principles may seem similar at first, they lead to very different formal and practical outcomes.

In the established framework of Equality of Opportunity (EOp), as delineated by pioneering researchers like Roemer (1998), an individual desirable outcome (y) is shaped by two key factors, *circumstances* (C), which are beyond a person's control, and *effort* (e), which reflects personal responsibility. Essentially, the model suggests that an individual's achievement can be explained by a combination of these two elements such that y = f(C, e). Equality of opportunity is achieved when differences in the outcomes due to circumstances, such as family background or socioeconomic status, are removed, while variations driven by individual effort are considered fair. For analytical convenience, both effort and elements within the circumstances vector are treated as discrete variables, allowing the population to be segmented into 'types', groups of individuals with the same circumstances, and 'tranches', with a shared degree of effort. In such a discrete framework, the population is depicted as a matrix [Xij] with n rows for the types and m columns for the tranches. The ex post approach to compensation tries to reduce inequalities between cells that have the same effort but different levels of outcome (Fleurbaey and Peragine (2013)). The ex ante approach to compensation identify inequality based only on information about circumstances and the types individuals belong to, before the effort is revealed.

4 Measuring ex-ante inequality of opportunity

4.1 Standard approaches and machine learning estimation

Scholars often estimate inequality of opportunity through a two-step process. First, a counterfactual distribution is estimated parametrically or non-parametrically, leaving only inequality due to circumstances. Then, an inequality index is applied to the counterfactual distribution. For a continuous outcome, Bourguignon et al. (2007) suggest quantifying the variability due to the vector of individual circumstances by Ordinary Least Squares (OLS):

$$y_i = C_i \Psi + \epsilon_i \tag{1}$$

where Ψ capture both the direct effect of circumstances and the indirect effect that circumstances play, through their effect on effort²(Brunori (2016)). Finally, the ex-ante counterfactual distribution is represented by the distribution of the predicted outcomes.

$$\tilde{Y}_{EA} = C\tilde{\Psi} \tag{2}$$

When the outcome is a binary variable, the observed distribution denoted by $[Y_i]$, can be first transformed into a counterfactual distribution $[\tilde{Y}_i^*]$ from the reduced-form regression of the binary outcome on circumstances, using a probit (Palmisano et al. (2022)) or a logit model. Then, in the second step, an inequality index is applied to the counterfactual distribution $[\tilde{Y}_i^*]$ of predicted probabilities, capturing only the inequality component attributable to unfair differences. Checchi and Peragine (2010) proposed to estimate nonparametrically the ex-ante inequality of opportunity (IOp) partitioning the population into types based

²However, it is important to note that this relationship holds only for linear models. In nonlinear models, the decomposition of effects does not hold exactly (Roemer and Trannoy (2016)). However, Jusot et al. (2013) found that the reduced form remains a reasonable approximation even in nonlinear contexts.

on all observable circumstances and measuring the inequality in the conterfactual vector obtained by replacing each income with its type mean income.

Machine learning algorithms overcome some limitations of traditional IOp approaches in the literature. Brunori et al. (2023) show that some machine learning techniques, in particular CIT and CIFs, outperform other methods in predicting IOp out-of-sample. Despite this capacity being practically valuable, it is rarely explicitly recognized as a primary objective or systematically evaluated in econometrics (Athey and Imbens (2019)). The main characteristic of supervised machine learning is that model specification is no longer arbitrarily imposed. Instead, the interactions between circumstances in shaping the outcome are determined by the algorithm's effort to maximize the explanation of the variability of the outcome. These algorithms are particularly adept at predicting the behavior of a dependent variable based on a set of observable independent variables.

CIFs is a part of the supervised machine learning techniques designed for predictive modeling (Breiman (2001); Hothorn et al. (2006)) that involves the generation of multiple trees derived by CIT algorithm, where the final outcome is an average of the predictions from these trees. The fundamental process of a CIT involves a sequence of steps which align well with Roemer's theoretical framework of inequality of opportunity. Despite lacking the straightforward interpretability of one single regression tree in terms of a society circumstances structure, CIFs generally offer superior performance in terms of prediction accuracy.

Brunori et al. (2023) suggest applying machine learning techniques to balance the risk of missing out key variables (downward bias) against the risk of overfitting (upward bias). As discussed by Carranza (2023), IOp measures obtained with these approaches can still be termed as the "lower bound" estimates of IOp, since they are based on a set of observable circumstances. In this paper, we adopt the ex ante utilitarian approach focusing on inequality between types.

4.2 Conditional Inference Forests algorithm

CIFs extend the CIT approach by building multiple trees and averaging their predictions to improve accuracy and stability with the following procedure:

- 1. Define a significance threshold 1α and the number of trees B^* .
- 2. Generate a random subsample $S' \subset S$ that contains approximately 60% of the observations. Repeat this process for each tree, constructing B^* trees in total.
- 3. At each splitting point, rather than considering all explanatory variables, randomly select a subset of circumstances of size P^* from the total available circumstances P. This reduces the risk of overfitting and ensures that there are diverse splitting variables between trees.
- 4. Each tree is generated by following these steps:
 - (a) Evaluate the correlation between the dependent variable and each candidate explanatory variable in the selected subset. If the Bonferroni-adjusted *p*-value exceeds the threshold α , terminate the process; otherwise, proceed.
 - (b) Determine the explanatory variable $[\omega^*]$ with the strongest association with the dependent variable, selecting it as the partitioning criterion.
 - (c) Identify the optimal binary split for the variable $[\omega^*]$, generating two sub-samples $[t_k, t_{-k}]$. For each potential partition, compute the *p*-value associated with the null hypothesis that the means of the two subsamples are equal.
 - (d) Choose the partition $[t_k, t_{-k}]^* = \arg \min_{T_{\omega^*}} p^{[t_m, t_{-m}]}$ that minimizes the *p*-value, ensuring the best statistical split.
 - (e) Repeat these steps iteratively for each sub-sample until the null hypothesis can no longer be rejected in any of the resulting groups.
- 5. Aggregate predictions from all B^* trees by averaging their outputs, thus stabilizing estimates and mitigating variance.

These modifications address the limitations of a single CIT. First, averaging over multiple trees mitigates the variance in the estimates and smooths the non-linear impact of circumstance characteristics. Second, the use of random subsets of all available circumstances increases the probability that all relevant explanatory variables will be identified as splitting variables at some point (Brunori et al. (2019a)). The ex-ante inequality of opportunity in is estimated as:

$$\hat{I}_{O_a} = I(\tilde{y}) \tag{3}$$

where \tilde{y} represents a counterfactual distribution obtained by replacing each individual's outcome with the population-weighted mean outcome of the type they belong to³.

4.3 Alpha tuning with K-fold cross-validation

Alternatively to specifying α a priori for each dataset, a 5-fold cross-validation is performed to determine its optimal value, defined as the one that maximizes the area under the receiver operating characteristic (ROC) curve (AUC). In binary classification, models typically output the predicted probability that an observation belongs to class 1, denoted as $\hat{P}(y = 1|X)$. To determine the predicted class, a decision threshold t must be set. An observation is classified as class 0 if $\hat{P}(y = 1|X) < t$ and as class 1 otherwise. The area under the ROC curve (AUC) is a numerical value ranging from 0 to 1 that quantifies a classifier's performance across different threshold values⁴ (James et al. (2013)). The ROC curve is plotted in a Cartesian plane, where the y-axis represents the true positive rate (TPR) and the x-axis represents the false positive rate (FPR). TPR or sensitivity measures the proportion of individuals who actually attained tertiary education among all positives and were identified by the model as such (eq.4). The False Positive Rate (FPR), on the other hand, measures the proportion of individuals who did not attain tertiary education but were incorrectly classified by the model among all negatives as having attained it⁵(Eq.5).

$$TPR = \frac{TP}{TP + FN} \tag{4}$$

$$FPR = \frac{FP}{FP + TN}$$
(5)

A higher TPR is desirable, while a lower FPR indicates better model performance. The ROC curve is constructed by computing TPR and FPR for a range of threshold values, typically sampled uniformly between 0 and 1. Each point on the ROC curve corresponds to a specific threshold t and represents the pair (TPR, FPR). The AUC is defined as the area

³Since the individual outcome is a binary variable (e.g. indicating whether an individual has attained tertiary education or not), the mean outcome within a type corresponds to the estimated probability that individuals belonging to that type have obtained a level of tertiary education.

⁴Since the outcome is binary, using AUC-ROC rather than MSE to assess model performance better captures how well the model distinguishes between positive and negative cases, rather than simply minimizing average prediction errors. MSE simply looks at average numerical errors, which can overlook whether the model actually makes good classification decisions.

 $^{{}^{5}}A$ true positive (TP) occurs when an individual who actually belongs to the positive class is correctly identified as such by the model. A false negative (FN) refers to the case in which a positive individual is incorrectly classified as negative. A false positive (FP) occurs when the model wrongly labels a negative individual as positive. A true negative (TN) describes the correct classification of an individual who truly belongs to the negative class.

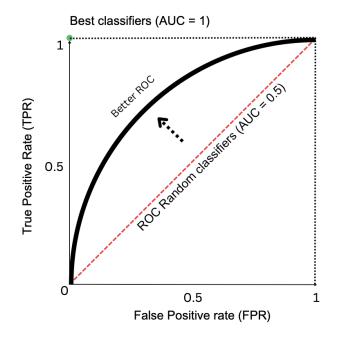


Figure 1: ROC curve

under this curve (Fig.1), representing the classifier's ability to distinguish between the two classes (Han (2022); Calì and Longobardi (2015)):

$$AUC = \int_0^1 ROC(t) dt = \frac{TPR \cdot FPR + \frac{TPR(TPR+1)}{2} - R}{TPR \cdot FPR}$$
(6)

where R represents the sum of ranks. An effective classifier should have an AUC significantly greater than 0.5, which represents the expected performance of a random classifier. We evaluate α over a predefined set of candidate values, in this case, {0.01, 0.05, 1}. For each value of α , we apply a 5-fold cross-validation as follows:

- 1. The data set is partitioned into five approximately equal-sized folds, denoted as D_1, \ldots, D_5 .
- 2. For each fold D_i $(i \in \{1, ..., 5\})$:
 - (a) Train the model using all folds except D_i .
 - (b) Generate predictions (probability estimates) for observations in the held-out fold D_i .
 - (c) Compute the AUC score for this fold, denoted as AUC_i , using the predicted probabilities and the corresponding true labels.

The cross-validated AUC for a given α is then calculated as the average AUC across all folds:

$$AUC(\alpha) = \frac{1}{5} \sum_{i=1}^{5} AUC_i$$
(7)

Finally, the optimal value of α used in CIFs is the one that maximizes the cross-validated AUC:

$$\alpha^* = \arg\max_{\alpha} AUC(\alpha) \tag{8}$$

4.4 Inequality of opportunity index

The reference inequality metric is a Dissimilarity index (D-index) estimated as follows:

$$D = \frac{1}{2N\bar{\hat{y}}} \sum_{i=1}^{N} \left| \hat{y}_i - \bar{\hat{y}} \right|$$
(9)

Essentially, it represents the average deviation of predicted outcomes \hat{y}_i from the mean prediction \bar{y} , as obtained non-parametrically through the CIFs algorithm. The interpretation of the D-index is similar to that of the Gini index: values close to 0 indicate low inequality of opportunity, while values approaching 1 reflect high inequality of opportunity. The Dindex provides point estimates of absolute inequality of opportunity, while the use of the relative measure is restricted to continuous outcome variables. This is because the inequality measure is uniformly applicable to both the actual outcome and the conditional outcome. Consequently, calculating the relative inequality of opportunity measure accurately becomes unfeasible in such scenarios (Juárez and Soloaga (2014)). Following the three-step procedure introduced by Barros et al. (2008), the D-index is also computed by deriving the conditional probabilities through a parametric approach⁶ to compare the model accuracy.

3. Compute the D-index as in equation 9.

 $^{^{6}}$ Barros et al. (2008) tree steps procedure:

^{1.} Estimate a probit model on whether individual *i* has achieved a tertiary education degree as a function of his or her circumstances $P(I = 1 | x_1, ..., x_m) = \Phi(\sum_{k=1}^m \beta_k x_k)$ where $\Phi(\cdot)$ denotes the cumulative distribution function (CDF) of the standard normal distribution, **x** is the vector of individual circumstances, and β is the vector of parameters.

^{2.} Given these coefficient estimates, obtain for each individual in the sample the predicted probability of achiving tertiary education, denoted as $\hat{p}_i = \Phi\left(\hat{\beta}_0 + \sum_{k=1}^m x_{ki}\hat{\beta}_k\right)$

5 Two-stage Data Envelopment Analysis

5.1 First Stage: DEA

DEA and Stochastic Frontier Analysis (SFA) represent two principal methodologies for assessing the efficiency in higher education. SFA, introduced by Aigner et al. (1977), belongs to parametric approaches, which specify the efficient production function based on microeconomic theory. The key advantage of SFA lies in its statistical robustness in the determination of efficiency scores. However, it the functional form of the production function is known and the distribution of efficiency scores is predetermined (Agasisti (2023)).

Conversely, DEA, a nonparametric approach, does not impose a predefined functional relationship between inputs and outputs but instead defines an efficiency frontier by enveloping observed data. This allows for greater flexibility and applicability to organizations that do not pursue profit maximization, such as public or non-profit institutions (Emrouznejad and Yang (2018)). Since the analysis covers multiple countries over three years, specifying a standard production function a priori might be suboptimal due to cross-country heterogeneity in educational systems. Therefore, a DEA model is chosen as the preferred methodology to evaluate efficiency.

DEA, originated by Farrell (1957) and revised by Charnes et al. (1978), is a mathematical programming tool that has gained widespread use for evaluating the performance of comparable units in using available resources to produce a set of outputs. The underlying assumption is that if one Decision Making Unit (DMU), say A, can produce a certain quantity of output Y(A), using a specific amount of input X(A), then other DMUs should be capable of achieving the same level of performance if they operate efficiently. The DMU with the highest performance is assigned an efficiency score of 1, while the other DMUs receive scores between 0 and 1 that reflect their performance relative to this benchmark. Although a DMU may be classified as fully efficient, this does not imply that there is no possibility for improvement; rather, it means that no other DMU performs better based on the available data, making this the best attainable performance level. Initially developed under the assumption of Constant Returns to Scale (CRS), the modifications to DEA for addressing Variables Returns to Scale (VRS) were introduced by Banker et al. (1984). CRS typically applies only to limited ranges and must be supported by evidence.

The choice of an output orientation in this application is informed by the nature of the higher education sector (Agasisti and Johnes (2015)). This approach aims to quantify the proportional increase in output required for the DMU to reach the efficiency frontier without increasing resources.

5.2 Second Stage: tobit model

The DEA models we have discussed so far operate under the assumption that all inputs and outputs are discretionary, meaning they are fully controlled by the management of the DMU. However, in the real world, some inputs or outputs may be beyond government control. This situation can be addressed with a combined approach of DEA and regression. The purpose of the regression analysis is to assess whether other exogenous factors significantly affect efficiency scores. Therefore, a tobit model is estimated to determine the nature and extent of this influence, by modeling the output efficiency scores as a function of potential factors Z.

5.3 Measuring technical efficiency in Higher Education

The efficiency of public expenditure can be derived from the following general function for each country.

$$Y_i = f(X_i), \quad i = 1, \dots, n \tag{10}$$

where Y_i represents a composite indicator for the output measure in country *i*, while X_i refers to the associated input.

The following linear programming problem (Coelli et al. (2005)) formulates the DEA model for an output-oriented approach, assuming variable returns to scale:

$$\max_{\substack{\phi,\lambda}} \phi$$
s.t. $-\phi y_i + Y\lambda \ge 0,$
 $x_i - X\lambda \ge 0,$
 $\mathbf{N1}'\lambda = 1,$
 $\lambda \ge 0.$

$$(11)$$

Suppose there are k inputs and m outputs for n countries, with x_i the input vector and y_i the output vector for country i. The matrices X and Y represent the inputs and outputs for all countries, with dimensions $k \times n$ and $m \times n$, respectively; $1 \leq \phi < \infty$ and $\phi - 1$ is the proportional increase in outputs that could be achieved by the country i, with input held constant. Note that $1/\phi$ defines an efficiency score (TE) which varies between zero and one. The vector λ , with dimension $n \times 1$, consists of weights that determine where an inefficient country would be positioned if it were to become efficient. These weights are associated with the peers of the inefficient country, which are other countries used as benchmarks due to their higher efficiency. The constraint $\mathbf{N1}'\lambda = 1$ ensures variable returns to scale (convexity of the frontier), where $\mathbf{N1}$ is a $N \times 1$ vector of ones. Without this constraint, constant

returns to scale would be assumed.

Since in the DEA we need to insert increasing outputs as the desired objective, and given that higher D-Index values imply higher inequality, the dissimilarity index requires a double transformation as follows.

• Step 1: Following Afonso et al. (2010), the inequality index is inverted to ensure that higher values are desirable:

$$D = 1 - \left(\frac{1}{2N\bar{\hat{y}}}\sum_{i=1}^{N} \left|\hat{y}_{i} - \bar{\hat{y}}\right|\right)$$
(12)

• Step 2: Multiply the inverted index by the share of graduates N_T to account for country differences. A low value of D only indicates that opportunities are equally distributed, but it does not provide any information about the absolute level of tertiary education attainment⁷:

$$D^* = \left[1 - \left(\frac{1}{2N\bar{\hat{y}}}\sum_{i=1}^N \left|\hat{y}_i - \bar{\hat{y}}\right|\right)\right] \times \frac{N_T}{N}$$
(13)

Without this adjustment, a low value of inequality of opportunity could be mistakenly interpreted as a positive signal, even in contexts where the completion of tertiary education is more residual. In this framework, the D^{*} indicator is adopted as the output variable, while public expenditure on tertiary education as the input.

Under the 1 input – 1 output specification, the efficiency score of a country can be estimated as the ratio of the observed output to the potential output if the country were on the efficient frontier. Figure 2 illustrates graphically the concept of technical efficiency under VRS assumption. Points A, B, C, and D lie on the VRS efficient frontier, representing DMUs that are technically efficient - producing the maximum achievable output for a given level of input. In contrast, points E and F are below the frontier and therefore are considered inefficient. In an output-oriented framework, the degree of inefficiency is assessed by projecting E and F on the VRS frontier, resulting in points E' and F', respectively. The vertical distance between each point and its projection reflects the output shortfall that could be eliminated if the DMU operated efficiently, assuming that the input levels remain unchanged.

⁷This approach is very similar to the procedure used to construct the Human Opportunity Index by Barros et al. (2008). However, in this paper, we multiply the inverted inequality index by the actual share of individuals with tertiary education, rather than by the coverage rate, that is, the average probability of access. The rationale for this choice is that the objective of the efficiency analysis is to maximize an indicator that accounts for the actual number of individuals attaining tertiary education. The resulting indicator therefore captures both the level of inequality and the actual number of people with higher education.

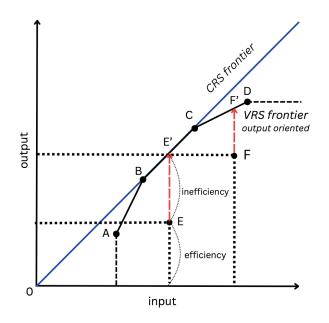


Figure 2: DEA frontier

6 Data and descriptive statistics

6.1 Outcome and circumstances

The European Social Survey (ESS) is a cross-national survey conducted across Europe since 2001. This paper relies on data collected in 2010 (ESS round 5)⁸, 2018 (ESS round 9)⁹ and 2023 (ESS round 11)¹⁰ as detailed in Table 1. The rationale behind this decision is based on methodological and contextual considerations. First, the ESS adopted the international classification of educational levels starting in 2010, enabling a standardized and robust comparison of educational levels across countries, which is essential to ensure consistency and reliability in cross-national and longitudinal analysis. Second, each year reflects a distinct socioeconomic setting with potentially significant implications for EIOp.

- The year 2010 captures the beginning of European sovereign debt crisis, during which several countries implemented austerity measures, including cuts to education budgets and social spending.
- The year 2018 represents a phase of relative economic stability and recovery from the previous crisis. This period offers a theoretically relevant benchmark to assess whether

⁸https://doi.org/10.21338/NSD-ESS5-2010

⁹https://doi.org/10.21338/NSD-ESS9-2018

¹⁰https://doi.org/10.21338/ess11-2023

Country	2010	2018	2023
AT (Austria)	1	1	1
BE (Belgium)	1	1	1
CH (Switzerland)	> > 	~~~~~~~~~	1
CZ (Czechia)	\checkmark	1	
DE (Germany)	\checkmark	1	1
EE (Estonia)	\checkmark	\checkmark	
ES (Spain)	\checkmark	\checkmark	>>>>> >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
FI (Finland)	1	\checkmark	\checkmark
FR (France)	1	1	1
GB (UK)	1	1	1
HU (Hungary)	1	1	1
IE (Ireland)	1	1	~
IL (Israel)	~		
IT (Italy)		\$ \$ \$ \$ \$ \$ \$ \$ \$	\ \ \ \ \ \ \ \
LT (Lithuania)	~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~		1
NL (Netherlands)	1		1
NO (Norway)	1	1	1
PL (Poland)	1	1	1
PT (Portugal)	1	~	~
RU (Russian Fed)	~	,	,
SE (Sweden)	~	~	~
BG (Bulgaria)	~	~	
DK (Denmark)	~	~	
HR (Croatia)	~	~	~
RS (Serbia)		~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~	\$ \$ \$
SK (Slovakia)	~	~	
UA (Ukraine)	\ \ \ \ \ \ \		\ \ \
GR (Greece)	~	1	*
CY (Cyprus)	× /	× /	× /
SI (Slovenia)	~	~	~
ME (Montenegro)		~	

Table 1: Country availability by survey round and year

Note: The checkmark (\checkmark) indicates that data are available for the respective country and year, with sufficient information for the estimates.

improved macroeconomic conditions and potential reinvestment in education systems translated into greater equality of educational opportunities.

• Finally, 2023 captures the post-pandemic context, following the COVID-19 crisis and the end of most emergency measures¹¹.

¹¹the data collection process for Round 10 (2020) was significantly disrupted by the COVID-19 pandemic, with nine countries shifting to a self-completion method and 22 countries continuing with face-to-face interviews. Furthermore, the sampling for round 10 occurred between 2020 and 2022, making the data less representative of the year 2020 and introducing inconsistencies in cross-country and time series comparisons; for this reason, the year 2018 is considered the pre-pandemic reference period.

This analysis focuses on individuals aged 25 to 70 with information on whether they have obtained a tertiary education degree. The outcome variable is a binary indicator, constructed by distinguishing between higher and lower levels of education (see Table 2). Specifically, we generate a dummy variable that aggregates all categories corresponding to tertiary education¹².

Regarding the circumstance variables (Tab.3), gender, parental education, and parental employment status when the respondent was 14 years old are included. The latter is used as a proxy for financial hardship, which may influence the probability of accessing tertiary education¹³. Parental education refers to the highest level of education that an individual has successfully completed. In the European Social Survey (ESS), this information is captured through a harmonized variable called EISCED, which standardizes national education levels in an internationally comparable format. Because respondents may struggle to interpret international education categories, this survey relies on country-specific questions to collect data on educational qualifications. However, after data collection, these country-specific responses are reclassified as a standardized cross-national variable for use in comparative research.

Value	Category	2010	2018	2023
1	ES-ISCED I, less than lower secondary	3474	1523	1097
2	ES-ISCED II, lower secondary	5501	4861	3432
3	ES-ISCED IIIb, lower tier upper sec.	6130	6250	4800
4	ES-ISCED IIIa, upper tier upper sec.	9292	8176	6512
5	ES-ISCED IV, advanced vocational	5548	4822	3426
6	ES-ISCED V1-V2, lower and higher tertiary	9466	10072	9258
55	Other	65	68	61

Table 2: Respondent Highest Level of Education, all countries per round – 2010, 2018, 2023

 $^{^{12}\}mathrm{A}$ value of one is assigned to individuals who have attained either lower or higher tertiary education, zero otherwise.

 $^{^{13}}$ To ensure comparability with parametric estimations (probit) and avoid collinearity problems, the original circumstance variables are recoded as in Tab.3.

Variable	Category	2010	2018	2023
Gender	Male	18081	16770	13431
	Female	21476	19066	15228
Father's education	less than lower secondary	12226	8320	6511
	secondary	17694	17950	14449
	advanced vocational	2658	2593	2164
	higher tertiary	3842	4146	3385
Mother's education	less than lower secondary	13790	9496	7189
	secondary	18501	19103	15431
	advanced vocational	2563	2274	1699
	lower tertiary	2729	3210	2811
Father's employment status at 14	Employee / Self-employed	34423	30905	25865
	Not working / absent	4062	3722	2049
Mother's employment status at 14	Employee/Self-employed	22828	21419	17575
	Not working / absent	16115	13689	10637

Table 3: Circumstances – 2010, 2018, and 2023, all countries per round

6.2 Public expenditure and exogenous factors

In general, the literature suggests that higher public spending on education and social protection is associated with lower levels of IOp in various domains, ranging from income to education. Marrero and Rodríguez (2012) find that expenditures on social exclusion, healthcare, and childcare are strongly and negatively correlated with both total inequality and IOp. Ferreira and Gignoux (2014) investigate the relationship between public spending on primary education and educational IOp. They find a significant negative correlation between the share of public educational expenditure allocated to primary schooling and IOp in reading and science. Checchi et al. (2016) find that public spending, particularly on preprimary education, is the only education-related variable that consistently retains statistical significance in relation to IOp, regardless of the model specification. Palmisano et al. (2022) investigate the determinants of EIOp in Europe. Their analysis shows that public spending on tertiary education, measured as a share of GDP or as a percentage of total government expenditure, is strongly and negatively correlated with EIOp in all model specifications. However, these findings should be interpreted as associative rather than causal.

The OECD database¹⁴ provides data on public spending on education, including direct expenditures by public entities on educational institutions, as well as education-related public subsidies allocated to households and managed by educational institutions. To investigate the relationship between public expenditure in tertiary education (% GDP) and EIOp, a pair-

¹⁴https://www.oecd.org/en/data.html

wise correlation analysis is performed across all available observations for 2010 and 2018¹⁵. The results in Figure 3, align with the existing literature, suggesting a negative relationship between public expenditure and inequality of opportunity, statistically significant at the 5% level only in 2018. While informative, these findings do not establish causality; instead, they provide indicative support for the theoretical relationship between input and output in the DEA framework¹⁶.

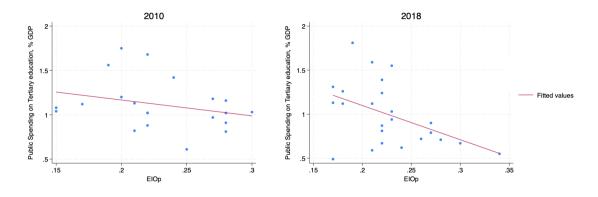


Figure 3: Pairwise correlations

It is reasonable to hypothesize that spending efficiency is shaped by factors such as the technological capabilities and skills within the public sector, the institutional framework, and the broader international constraints faced by a country. All contextual variables used in the second stage are sourced from the OECD database. Gross Domestic Product (GDP) is a key economic indicator, representing the total value added from the production of goods and services within a country over a specific period. This measure is based on nominal GDP at current prices (US dollars). Another critical factor in the analysis is the quality of education, which is proxied by the student-to-teacher ratio, available only for 2018. This ratio is computed as the total number of full-time equivalent students enrolled at a given education level divided by the total number of full-time equivalent teachers at the same level. In addition, the relative unemployment rate by educational level is incorporated to assess labor market outcomes based on educational attainment. This indicator measures the percentage of unemployed people aged 25-64 years in the total labor force of the same age group. Moreover, primary to post-secondary (non-tertiary) public spending on education as a percentage of GDP is included to support the hypothesis that that additional investments throughout the entire educational ladder can enhance EIOp. A detailed description of these indicators is provided in Tables 4 and 5.

 $^{^{15}\}mathrm{Public}$ spending and other data are not available for 2023 in the OECD database. For the same reason, DEA will not be performed for 2023

¹⁶While DEA is a non-parametric approach that does not require statistical significance for inference, this preliminary correlation analysis provides contextual support for the underlying relationship.

Country	Input	GDP	$\mathbf{SpendSec}$	Unemplter	Unemplsec	TerShare	RelUnemp
BE	1.20	39844	4.14	4.02	6.58	0.38	0.61
CZ	0.88	28042	2.43	2.52	6.17	0.11	0.40
DE	1.02	40469	2.91	3.09	6.92	0.20	0.44
\mathbf{EE}	1.04	21587	3.75	9.19	17.82	0.28	0.51
ES	1.03	31818	2.97	10.37	17.17	0.27	0.60
\mathbf{FI}	1.75	38950	3.91	4.41	7.48	0.27	0.58
\mathbf{FR}	1.18	36066	3.60	4.93	7.20	0.17	0.68
IE	1.16	43221	4.27	7.05	13.84	0.20	0.50
NL	1.12	45307	3.28	2.28	3.07	0.27	0.74
NO	1.56	58223	4.87	1.57	2.17	0.40	0.72
PL	1.02	20993	3.39	4.19	8.89	0.26	0.47
CH	1.13	54334	3.12	3.06	4.32	0.19	0.70
PT	0.91	27295	3.74	6.3	9.7	0.11	0.64
SE	1.42	41961	3.72	4.48	6.44	0.27	0.69
\mathbf{SI}	1.02	27579	3.49	4.07	6.93	0.18	0.58
DK	1.68	43045	4.55	4.56	6.3	0.36	0.72
IL	0.82	29467	3.59	4.18	6.8	0.33	0.61
LT	1.08	19828	3.27	6.79	20.61	0.27	0.32
RU	0.97	22008	1.85	4.26	8.74	0.32	0.48
SK	0.61	25204	2.59	4.84	12.25	0.17	0.39
HU	0.81	21693	2.71	4.13	9.49	0.17	0.43

Table 4: Country-level indicators used in DEA analysis - 2010

Notes: input is public spending on tertiary education (% of GDP); GDP is Nominal gross domestic product (US dollars per capita, current PPPs); SpendSec is public spending on primary to post-secondary non-tertiary education (% of GDP); Unemplter is unemployment rate on tertiary education (% of 25-64 year-olds); Unemplsec is unemployment rates on Upper secondary, non-tertiary education (% of 25-64 year-olds); TerShare denotes the share of individuals who have attained a tertiary education degree, estimated for each country in the dataset; RelUnemp is the relative unemployment rate, defined as the ratio of Unemplter to Unemplsec.

Country	Input	Quality	GDP	Unemplter	Unemplsec	$\mathbf{SpendSec}$	TerShare	RelUnemp
BE	1.26	20.99	52467	3.16	5.09	3.98	0.42	0.62
CZ	0.90	15.02	42016	1.14	1.9	2.68	0.18	0.6
DE	1.03	11.97	56273	1.87	2.85	2.63	0.29	0.65
\mathbf{EE}	1.12	12.85	37201	3.57	5.24	2.95	0.35	0.68
\mathbf{ES}	0.81	12.3	41074	8.43	13.79	2.61	0.27	0.61
\mathbf{FI}	1.39	15.33	49243	4.16	7.09	3.58	0.39	0.58
\mathbf{FR}	1.12	16.31	46398	5.03	8.25	3.36	0.24	0.60
IE	0.59	20.37	86434	3.52	5.34	2.16	0.31	0.65
NL	1.13	14.63	58818	3.36	3.89	3.02	0.39	0.86
NO	1.81	9.35	70253	2.2	2.92	4.63	0.49	0.75
$_{\rm PL}$	0.94	13.85	32361	1.8	3.65	2.79	0.28	0.49
CH	1.24	13.12	70656	3.52	4.41	3.09	0.25	0.79
\mathbf{PT}	0.67	14.3	34725	4.73	6.57	3.31	0.29	0.71
SE	1.31	10.14	53122	3.53	3.37	3.97	0.38	1.04
SI	0.87	14.43	38656	3.56	5.28	2.86	0.25	0.67
AT	1.55	13.77	56655	2.97	3.83	2.83	0.14	0.77
BG	0.67	11.5	24053	-	-	1.89	0.23	-
DK	1.59	15.76	57231	4.03	3.46	3.7	0.44	1.16
GB	0.49	15.38	47212	2.17	3.07	3.39	0.33	0.70
$_{\rm HR}$	0.79	12.52	29045	-	-	2.5	0.16	-
HU	0.72	11.54	32258	1.34	3.03	2.54	0.16	0.44
IT	0.55	20.31	43583	5.73	8.43	2.92	0.16	0.67
LT	0.71	14.37	36492	2.59	7.97	2.21	0.27	0.32
SK	0.62	11.43	31514	2.84	5.44	2.49	0.19	0.52

Table 5: Country-level indicators used in DEA analysis - 2018

Notes: Indicators are the same as those defined in Table 4; quality of education, not available for 2010, is defined as the number of students per teaching staff at the tertiary level.

7 Results

7.1 Inequality of Opportunity in Tertiary education

Table 6 presents point estimates of EIOp obtained using the CIFs algorithm. The results indicate that CIFs outperform both the parametric and CIT approaches in almost all cases¹⁷. In contrast, CIT algorithm does not outperform the probit model ¹⁸. These findings align with the results of Perlich et al. (2003), who obtain the result that logistic regression tends to perform better with smaller datasets (fewer than 2,000 observations), whereas decision tree induction is more effective with larger samples. The average AUC exceeds 0.70, well above the 0.5 threshold for acceptable discrimination, indicating that the CIFs offers reliable predictive power and does not behave like a random classifier. Relative to the CIFs approach, the parametric method and CIT appear to overestimate the extent of inequality of opportunity. As a result, the choice of estimation method could have substantial implications for cross-country comparisons. Therefore, adopting more precise estimation techniques, such as CIFs, is crucial to ensure that country rankings accurately reflect EIOp in a way that best approximates reality. Point estimates for 2010, 2018, and 2023 (Tab.4) highlight significant cross-country disparities and a general declining trend over time in most cases. In general, apparent macroregional disparities emerge from these estimates. Northern European countries consistently report low levels of EIOp. Estonia and Lithuania recorded the lowest levels of EIOp in 2010, with a D-index of 0.15. In 2018, similarly low values were observed in the United Kingdom, Ireland, and Sweden (EIOp = 0.17). More recently, Finland registered the lowest value in 2023, with an EIOp point estimate of 0.15. Their well-established social safety nets and inclusive education systems may have contributed to more equitable outcomes, where socioeconomic background plays a relatively minor role in determining who attends and completes higher education. In contrast, Southern Europe continues to face higher levels of EIOp. Weaker welfare systems and ongoing economic instability can exacerbate these inequalities, limiting the effectiveness of reforms to reduce inequality of opportunity. The highest values were observed in Austria in 2010 (EIOp = 0.35), Italy in 2018 (EIOp = 0.34), and Croatia in 2023 (EIOp = 0.31), suggesting a persistent and substantial influence of individual circumstances on educational attainment in these contexts.

Despite persistent disparities between northern and southern European countries, variations over time (Fig.4) indicate that for almost all countries a reduction in estimated levels of EIOp is observed between 2010 and 2023¹⁹, suggesting a general improvement. Although

 $^{^{17}\}mathrm{Appendix}$ A.1 reports an example of a tree produced using CIT.

¹⁸The CIFs algorithm outperforms the CIT in all cases except one. The probit model (or parametric approach) performs better than CIFs in only 24 out of 78 cases. Further details are provided in the supplementary material.

¹⁹Variations are computed exclusively for those countries with available estimates for both reference years.

Country		20	010			20	018			20)23	
	EIOp	οα	AUC	N	EIOI	οα	AUC	N	EIO	οα	AUC	Ν
BE	0.20	0.05	0.76	1906	0.18	0.05	0.75	1091	0.16	0.05	0.76	935
BG	0.24	0.01	0.76	1636	0.22	0.05	0.75	1198	_	_	_	_
CH	0.21	0.05	0.73	1006	0.22	0.01	0.74	1028	0.20	0.05	0.75	912
CY	0.27	0.05	0.78	788	0.29	0.01	0.80	541	0.23	0.01	0.78	450
CZ	0.22	0.05	0.68	1742	0.27	0.05	0.76	1689	_	_	_	_
DE	0.28	1.00	0.69	1912	0.23	1.00	0.70	1463	0.20	0.05	0.72	1394
DK	0.22	0.05	0.73	1102	0.21	1.00	0.72	1031	_	_	_	_
\mathbf{EE}	0.15	0.01	0.70	1011	0.18	1.00	0.67	1245	_	_	_	_
ES	0.30	1.00	0.74	1309	0.22	0.05	0.72	1089	0.19	0.01	0.71	1237
\mathbf{FI}	0.20	0.01	0.71	1295	0.22	1.00	0.72	1186	0.14	0.01	0.70	1003
\mathbf{FR}	0.27	0.01	0.73	1098	0.21	0.01	0.71	1115	0.22	0.05	0.73	993
GB	0.29	1.00	0.74	1083	0.17	0.05	0.73	1159	0.16	0.01	0.75	725
GR	0.24	0.05	0.74	1945	_	_	_	_	0.20	0.05	0.71	2098
HR	0.30	0.05	0.79	1035	0.27	0.05	0.74	1271	0.31	0.05	0.77	1035
HU	0.28	0.01	0.72	1081	0.24	0.05	0.73	1075	0.24	1.00	0.73	1479
IE	0.28	0.01	0.76	1671	0.21	0.01	0.75	1436	0.22	1.00	0.73	1341
IL	0.21	0.01	0.72	1296	_	_	_	_	_	_	_	_
LT	0.15	0.01	0.66	895	0.28	1.00	0.74	1094	0.24	0.05	0.80	853
NL	0.17	0.01	0.74	1248	0.17	0.05	0.73	1002	0.15	0.05	0.69	1051
NO	0.19	0.05	0.72	1102	0.19	1.00	0.71	923	0.15	1.00	0.70	886
PL	0.22	0.01	0.77	1140	0.23	1.00	0.73	946	0.17	0.05	0.73	959
\mathbf{PT}	0.28	0.01	0.72	1277	0.30	1.00	0.75	671	0.26	0.01	0.73	901
RU	0.27	1.00	0.74	1516	_	_	_	_	_	_	_	_
SE	0.24	1.00	0.71	923	0.17	0.05	0.71	960	0.17	0.05	0.74	750
\mathbf{SI}	0.22	0.05	0.73	936	0.22	0.05	0.73	888	0.21	0.05	0.71	805
SK	0.25	0.05	0.71	1347	0.24	1.00	0.69	754	0.19	1.00	0.70	1040
UA	0.19	0.05	0.74	1165	_	_	_	_	_	_	_	_
AT	0.35	1.00	0.75	1440	0.23	0.01	0.71	1763	0.23	0.05	0.71	1573
IT	_	_	_	_	0.34	0.05	0.78	1751	0.28	0.05	0.77	1739
ME	_	_	_	_	0.23	0.01	0.74	912	_	_	_	_
RS	_	_	_	_	0.33	0.05	0.79	1440	0.27	1.00	0.77	1044

Table 6: EIOp, α , AUC, and observations (N) by Country and Year

Note: EIOp point estimates are derived using CIFs algorithm removing missing values with the following parameters: the number of trees is set to 150 (ntree = 150), the number of variables randomly selected at each split is 3 (mtry = 3), and the statistical test employed is a Bonferroni-adjusted quadratic test (testtype = "Bonferroni", teststat = "quad"). The minimum criterion for a split is defined as $1 - \alpha$ cross validated, and the minimum number of observations in each terminal node is set to 10 (minbucket = 10). All other parameters are kept at their default values. Robustness checks are conducted using 300 and 500 trees, yielding consistent results. Probit and CIT estimates are available in the additional material.

causality cannot be inferred from this analysis, the trend may be consistent with the hypothesis that successful interventions to expand access and attainment of higher education for disadvantaged groups have contributed to a gradual leveling of the playing field. However, Croatia (HR) and Lithuania (LT) are the exceptions, as EIOp increased slightly during the same period. To provide an overview of how the COVID-19 pandemic may have influenced EIOp, the variation between the pre-pandemic period (2018) and the post-pandemic period (2023) is examined. This comparison (Fig.5) reveals that EIOp has decreased slightly in most countries or remained stable, with the exception of France, Croatia, and Ireland. Such findings may seem unexpected at first, but several plausible factors can explain these patterns. First, many governments implemented targeted policies during the pandemic, such as financial aid, digital learning initiatives, and support programs for disadvantaged people, which may have enabled students to continue their studies and temporarily lowered structural barriers (Farnell et al. (2021)). Second, the pandemic affected not only the most vulnerable groups, but also traditionally advantaged students, potentially leading to a general leveling of educational outcomes and, consequently, a marginal decline in EIOp. Third, some countries revised their admission criteria for higher education during the pandemic, suspending standardized exams or adopting alternative systems, which may have reduced the influence of family background on access to higher education. However, the most severe consequences for educational inequality might emerge in the years ahead, as the cohorts most affected by limited educational resources reach the age of higher education entry. Tertiary education is typically completed by 24, so the effects of policy changes are often felt most strongly by specific age cohorts (Palmisano et al. (2022)). For example, reforms that reduce tuition fees or expand financial aid will likely benefit younger cohorts still deciding whether to pursue higher education. Conversely, although distance learning modalities were implemented in many countries, some students in primary and secondary education, particularly those living in disadvantaged areas, may not have had access to the necessary digital tools, potentially compromising their educational trajectories in the long term.

The patterns emerging from the estimates suggest that changes in EIOp are embedded within broader institutional, demographic, and policy dynamics that evolve over time and interact with country-specific contexts. As such, although the methodological improvements used in the analysis allow for a more accurate capture of these dynamics, interpreting crosscountry differences requires caution, particularly when shifts in EIOp may reflect not only structural reforms, but also transitory responses to external shocks.

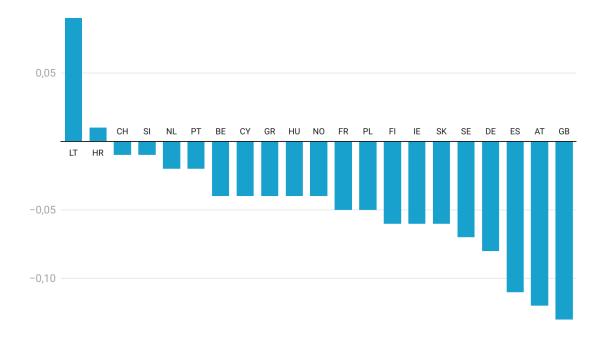
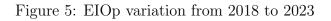
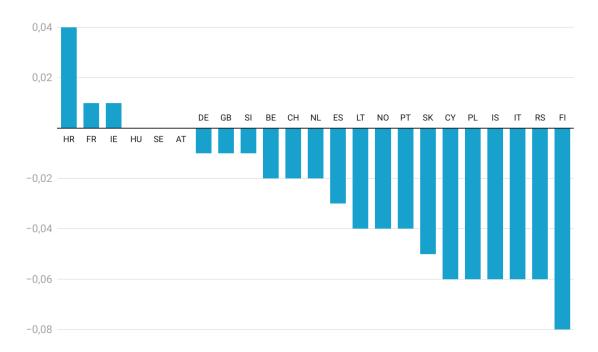


Figure 4: EIOp variation from 2010 to 2023





7.2 Two stage Data Envelopment Analysis results

7.3 First stage: DEA model

Tables 7 and 8 show VRS output-oriented DEA efficiency scores from the one input and one output specification for 2010 and 2018. Although efficiency scores fluctuate over time, it is essential to recognize that the inclusion or exclusion of specific countries in each year of the analysis inevitably influences the efficiency frontier and the corresponding relative scores. Consequently, observed variations over time may reflect changes in sample composition rather than genuine improvements or reductions in efficiency. To overcome this limitation, the analysis is limited to the panel component.

The results for 2010 indicate that Belgium (BE), Norway (NO) and Slovakia (SK) are fully efficient, serving as benchmarks for other countries. The average efficiency score is 0.69, suggesting that, maintaining constant the level of public spending on tertiary education, it could theoretically be possible to improve the output by approximately 31% for countries in the sample. Portugal (PT) has the lowest efficiency score, indicating that resource allocation is less efficient compared to the most efficient performers.

In 2018, the average efficiency score increases to 0.78, implying that an efficiency gap of 22% remains, but with a 0.09 efficiency improvement. Belgium (BE), Norway (NO), and Ireland (IE) achieve full efficiency, meaning that their public spending strategies effectively translate into more equitable educational outcomes. Hungary and Czechia exhibit the lowest efficiency scores, reflecting persistent inefficiencies in the allocation of higher education resources.

Since DEA does not inherently provide a way to statistically evaluate whether a DMU's deviation from the frontier is significant, it is crucial to test the sensitivity. If a DMU is truly fully efficient, it should serve as a peer for nearby inefficient units. If it fails to serve as a peer for any inefficient DMU, its efficiency may be questionable. A country can have one or two peers, depending on its projection onto the efficient frontier.

The results in Tables 7 and 8 are consistent with previous explanation; the efficient countries consistently serve as peers for inefficient ones. This confirms their position on the efficient frontier, as these countries not only achieve the best outcomes in using public spending on tertiary education but also act as benchmarks for less efficient countries.

Country	Input	Output	TE	Peer1	Peer2
BE	1.20	30.4	1.00	BE	NA
CZ	0.88	8.58	0.41	BE	SK
DE	1.02	14.4	0.57	BE	SK
\mathbf{EE}	1.04	23.8	0.92	BE	SK
\mathbf{ES}	1.03	18.9	0.74	BE	SK
FI	1.75	21.6	0.66	NO	NS
\mathbf{FR}	1.18	12.41	0.41	BE	SK
IE	1.16	14.4	0.49	BE	SK
NL	1.12	22.41	0.80	BE	SK
NO	1.56	32.4	1.00	NO	NA
$_{\rm PL}$	1.02	20.28	0.81	BE	SK
CH	1.13	15.01	0.53	BE	SK
\mathbf{PT}	0.91	7.92	0.36	BE	SK
SE	1.42	20.52	0.64	BE	SK
SI	1.021	14.04	0.56	BE	SK
DK	1.68	28.08	0.85	NO	NA
LT	1.08	22.95	0.85	BE	SK
SK	0.61	12.75	1.00	SK	NA
HU	0.81	12.24	0.48	BE	\mathbf{SK}
Average	1.13	18.54	0.69		

Table 7: Technical efficiency (TE) and reference peers – Panel Component - 2010

Note: Peer1 and Peer2 indicate the most efficient reference countries used in the efficiency comparison.

Table 8: Technical efficiency (TE) and reference peers – Panel Component - 2018

Country	Input	Output	TE	Peer1	Peer2
BE	1.26	34.44	1.00	BE	NA
CZ	0.90	13.14	0.45	BE	IE
DE	1.03	22.33	0.71	BE	IE
\mathbf{EE}	1.12	28.7	0.88	BE	IE
\mathbf{ES}	0.81	21.06	0.75	BE	IE
\mathbf{FI}	1.39	30.42	0.85	BE	NO
\mathbf{FR}	1.12	18.96	0.58	BE	IE
IE	0.59	24.49	1.00	IE	NA
NL	1.13	32.37	0.99	BE	IE
NO	1.81	39.69	1.00	NO	NA
PL	0.94	21.56	0.72	BE	IE
CH	1.24	19.5	0.57	BE	IE
\mathbf{PT}	0.67	20.3	0.79	BE	IE
SE	1.31	31.54	0.90	BE	NO
SI	0.87	19.5	0.68	BE	IE
DK	1.59	34.76	0.92	BE	IE
HU	0.72	11.84	0.44	BE	IE
LT	0.71	19.44	0.74	BE	IE
SK	0.62	14.44	0.57	BE	IE
Average	0.96	25.73	0.78		

Note: Peer1 and Peer2 indicate the most efficient reference countries used in the efficiency comparison.

7.4 Second stage: Tobit model

The purpose of the Tobit analysis is to assess whether Z environmental factors, also known as non-discretionary or exogenous inputs affect efficiency scores and if so, to determine the nature and extent of this influence.

The results of the Tobit analysis in 2018 (Tab.9) show that public expenditure on primary and secondary education significantly influences efficiency scores, highlighting the indirect impact of broader educational investments on equality and completion in tertiary education. Although they are not statistically significant, the other variables exhibit the expected signs of the coefficients²⁰. GDP per capita positively affects output efficiency scores, indicating that wealthier countries tend to utilize public spending more efficiently in tertiary education. Furthermore, student-to-teacher ratio has a positive impact on efficiency, suggesting that higher quality in educational resources increases spending efficiency. The relative unemployment rate (tertiary / secondary) negatively affects efficiency scores, as a higher value indicates that tertiary education fails to provide a distinct labor market advantage over secondary education. The negative coefficient reflects the notion that, when higher education does not lead to better employment prospects relative to lower educational levels, the returns to public investment are diminished, thus reducing the overall efficiency of the education system. Similar results are observed for 2010 with coefficient signs aligning with expectations (Tab.10). However, public spending on primary and secondary education does not reach conventional levels of statistical significance, although it approaches the 10%threshold. Furthermore, data on quality of education are not available for 2010 in the OECD database. These findings should not be interpreted as evidence of causality, but rather as simple associations between efficiency scores and contextual factors, offering preliminary insight into why certain countries require greater resources to achieve comparable levels of output.

²⁰The Tobit regression is presented for illustrative purposes only. The analysis is based on a limited number of observations, which considerably reduces the statistical power of the model. As a result, the findings should be interpreted with caution and not considered conclusive. The primary objective is to provide preliminary information on the potential direction of the relationships, rather than to draw definitive inferences.

Variable	Coefficient	Std. Error	p-value
Quality	0.0197	0.0143	0.189
GDP	0.00000531	0.00000368	0.170
Primary to secondary spending	0.1394	0.0729	0.075^{*}
Relative unemployment rate	-0.0009	0.1797	0.996
Variance of TE	0.0225	0.0072	

Table 9: Tobit regression results - 2018

Note. The model includes 19 observations. Dependent variable: TE scores (bounded between 0 and 1). * indicates significance at the 10% level.

Variable	Coefficient	Std. Error	p-value
Quality	na	na	na
GDP	0.00000188	0.00000740	0.803
Primary to secondary spending	.1563193	.1100455	0.175
Relative unemployment rate	6353424	.7627627	0.417
Variance of TE	.0473	.01664	

Table 10: Tobit regression results - 2010

Note. The model includes 19 observations. Dependent variable: TE scores (bounded between 0 and 1).

7.5 Robustness checks

7.6 Global accuracy

Since final predictions do not explicitly classify individuals into binary categories, but instead assign a probability score, a default classification threshold (typically 0.5) is applied to generate predicted labels. This allows for the construction of a confusion matrix which summarizes the model's performance in distinguishing between those who have attained tertiary education (1) and those who have not (0).

Although the AUC-ROC metric used to perform a 5-fold cross-validation and select the optimal value of α gives back a measure of accuracy, the confusion matrix is computed as an additional robustness check. This enables the examination of specific types of misclassification errors made by the model and to ensure that its predictive performance remains reliable under a predefined classification threshold.

Table 11 aggregates the four components across all countries and years to derive an overall measure of accuracy, as defined in Equation 14.

Accuracy =
$$\frac{TP + TN}{TP + TN + FP + FN} = \frac{10515 + 61423}{10515 + 61423 + 4331 + 16439} \approx 0.77$$
 (14)

Table 11: Confusion matrix

	Predicted Positive (1)	Predicted Negative (0)
Actual Positive (1)	10515 (TP)	16439 (FN)
Actual Negative (0)	4331 (FP)	61423 (TN)

The model achieves an accuracy of 77%, indicating that it correctly classifies most observations and produces meaningful predictions rather than acting as a random classifier.

7.7 Sample size

As a further robustness check, the number of observations within each country is equalized over time to ensure that EIOp estimates are not driven by fluctuations in sample size. For each country-year pair, a random subsample is drawn corresponding to the minimum number of observations available for that country across the entire time span. A stratified random sampling strategy is implemented to mitigate potential class imbalance in the dependent variable. This procedure yields a balanced sub-sample with respect to the outcome classes, thereby minimizing biases that may result form disproportionate class distributions during model training. The results reveal no substantial changes in the point estimates; countries with the highest and lowest levels of inequality of opportunity remain at the top and bottom of the ranking. All estimates in the main and in this robustness analyses are obtained after removing missing values, that is, by dropping all rows in the dataset with any missing observations. Although CIFs algorithm is capable of handling missing values by searching for alternative splitting points, doing so tends to reduce its predictive advantage over trees and probit regressions. However, results obtained with and without missing values yield very similar estimates.

7.8 Unbalanced DEA analysis

The DEA analysis is replicated by including all countries with available data for 2010 and 2018. Although technical efficiency scores are recognized to vary depending on the set of DMUs used to construct the production frontier, this approach allows the maximization of available information for each period. Consequently, efficiency estimates reflect the relative performance of countries within each year, based on the specific frontier defined by the corresponding sample. The results show that, despite changes in the composition of DMUs across both years, Norway consistently lies on the efficiency frontier²¹ with a technical efficiency scores that reflect different sample composition (Appendix A.2).

²¹Appendix A.3 shows the Production Possibility frontiers for balanced and unbalanced DEA

8 Conclusion

The paper assesses changes in inequality of opportunity and the efficiency of public spending in tertiary education across 31 countries, drawing on data from the European Social Survey for the years 2010, 2018, and 2023. EIOp point estimates highlight significant regional disparities, with northern European countries reporting the lowest point estimates, while high levels persist in southern Europe. This suggests that socioeconomic background continues to limit access to higher education. In terms of public spending efficiency in tertiary education, Norway and Belgium consistently lie on the efficient frontier, whereas countries such as Hungary, Italy, and Portugal register notably lower efficiency scores.

The changes over time from the pre-pandemic to the post-pandemic period reveal a reduction in inequality of opportunity in almost all countries. This may be attributed to the successful implementation of effective measures by governments. However, adverse effects could emerge in the longer term for younger cohorts. The average technical efficiency increased by 9% between 2010 and 2018, indicating progress in the ability of countries to convert public spending into higher graduates and more equal opportunities. Despite this overall improvement, Norway and Belgium continue to set the standard, consistently forming the efficient frontier.

A key limitation of this analysis is the available set of circumstance variables in the data, leading to lower bound estimates of EIOp. Although CIFs consistently outperform alternative methods implemented in this study, circumstances represent only a subset of all possible factors influencing educational inequality. As such, our estimates provide a limited view of the true extent of EIOp. Expanding the set of circumstances, if possible, would provide a more comprehensive picture.

A second caveat concerns the DEA approach; while it provides a robust method for benchmarking countries, it does not account for the long-term endogeneity of exogenous factors. For example, while quality of education may be treated as an exogenous variable in the short term, it is likely to become endogenous over time. Changes in educational policies, such as improvements in teacher training or infrastructure, can directly influence the quality of education. Thus, DEA efficiency scores should be interpreted with caution, particularly in the context of long-term policy evaluations.

Despite these limitations, two policy implications arise. First, efforts to reduce EIOp should continue to address structural barriers related to socioeconomic background, as persistent disparities suggest that individual circumstances still play a significant role in shaping access to and success in tertiary education.

The second policy implication suggests that achieving a more equitable distribution of opportunities and results in tertiary education does not necessarily require increased expenditure; rather, better results could be achieved through more efficient use of existing resources. However, the efficiency results reflect heterogeneous realities and should be considered as theoretical indications. Exogenous factors, such as the indirect effects of spending in primary and secondary education, may influence efficiency.

Moving toward more equitable higher education systems requires going beyond current limitations. Expanding the range of observed circumstances and gaining a deeper understanding of how educational quality contributes to social inequalities are essential next steps. Equally important is adopting a longer-term perspective to fully capture the effects of public policies and investment decisions over time. Efficiency should remain a central focus: improving how resources are used can have an impact equal to, if not greater than, simply increasing their volume. Only by embracing these broader and more refined approaches can we support policies that not only reduce inequality in the present, but also lay the foundations for fairer, more inclusive educational systems in the years to come.

Supplementary Information

Additional material is available online

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Disclosure of interest

The author reports there are no competing interests to declare.

References

- Afonso, A., Schuknecht, L., and Tanzi, V. (2010). Income distribution determinants and public spending efficiency. *The Journal of Economic Inequality*, 8(3):367–389.
- Agasisti, T. (2011). Performances and spending efficiency in higher education: a european comparison through non-parametric approaches. *Education Economics*, 19(2):199–224.

- Agasisti, T. (2023). The efficiency of higher education institutions and systems. In Handbook on Public Sector Efficiency, pages 274–290. Edward Elgar Publishing.
- Agasisti, T. and Johnes, G. (2015). Efficiency, costs, rankings and heterogeneity: the case of us higher education. *Studies in Higher Education*, 40(1):60–82.
- Aigner, D., Lovell, C. K., and Schmidt, P. (1977). Formulation and estimation of stochastic frontier production function models. *Journal of econometrics*, 6(1):21–37.
- Athey, S. and Imbens, G. W. (2019). Machine learning methods that economists should know about. *Annual Review of Economics*, 11(1):685–725.
- Aubyn, M. S., Garcia, F., Pais, J., et al. (2009). Study on the efficiency and effectiveness of public spending on tertiary education. Technical report, Directorate General Economic and Financial Affairs (DG ECFIN).
- Banker, R. D., Charnes, A., and Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management science*, 30(9):1078– 1092.
- Barros, R. P., Vega, J. M., and Saavedra, J. (2008). Measuring inequality of opportunities for children. *Washington, DC: World Bank*.
- Bossert, W. (1995). Redistribution mechanisms based on individual characteristics. *Mathe*matical Social Sciences, 29(1):1–17.
- Bourguignon, F., Ferreira, F. H., and Menéndez, M. (2007). Inequality of opportunity in brazil. *Review of income and Wealth*, 53(4):585–618.
- Breiman, L. (2001). Random forests. *Machine learning*, 45:5–32.
- Brunori, P. (2016). How to measure inequality of opportunity: A hands-on guide.
- Brunori, P., Hufe, P., and Mahler, D. (2023). The roots of inequality: Estimating inequality of opportunity from regression trees and forests. *The Scandinavian Journal of Economics*, 125(4):900–932.
- Brunori, P., Palmisano, F., and Peragine, V. (2019a). Inequality of opportunity in subsaharan africa. *Applied Economics*, 51(60):6428–6458.
- Brunori, P., Peragine, V., and Serlenga, L. (2012). Fairness in education: The italian university before and after the reform. *Economics of Education Review*, 31(5):764–777.

- Brunori, P., Peragine, V., and Serlenga, L. (2019b). Upward and downward bias when measuring inequality of opportunity. *Social Choice and Welfare*, 52:635–661.
- Calì, C. and Longobardi, M. (2015). Some mathematical properties of the roc curve and their applications. *Ricerche di Matematica*, 64:391–402.
- Carranza, R. (2023). Upper and lower bound estimates of inequality of opportunity: A cross-national comparison for europe. *Review of Income and Wealth*, 69(4):838–860.
- Charnes, A., Cooper, W. W., and Rhodes, E. (1978). Measuring the efficiency of decision making units. *European journal of operational research*, 2(6):429–444.
- Checchi, D. and Peragine, V. (2010). Inequality of opportunity in italy. *The Journal of Economic Inequality*, 8:429–450.
- Checchi, D., Peragine, V., and Serlenga, L. (2016). Inequality of opportunity in europe: Is there a role for institutions?. In *Inequality: Causes and consequences*, pages 1–44. Emerald Group Publishing Limited.
- Coelli, T. J., Rao, D. S. P., O'donnell, C. J., and Battese, G. E. (2005). An introduction to efficiency and productivity analysis. springer science & business media.
- Dipierro, A. R. and De Witte, K. (2024). The underlying signals of efficiency in european universities: a combined efficiency and machine learning approach. *Studies in Higher Education*, pages 1–20.
- Emrouznejad, A. and Yang, G.-l. (2018). A survey and analysis of the first 40 years of scholarly literature in dea: 1978–2016. *Socio-economic planning sciences*, 61:4–8.
- Farnell, T., Skledar Matijevic, A., and Scukanec Schmidt, N. (2021). The Impact of COVID-19 on Higher Education: A Review of Emerging Evidence. Analytical Report. ERIC.
- Farrell, M. J. (1957). The measurement of productive efficiency. Journal of the royal statistical society series a: statistics in society, 120(3):253–281.
- Ferreira, F. H. and Gignoux, J. (2014). The measurement of educational inequality: Achievement and opportunity. The World Bank Economic Review, 28(2):210–246.
- Ferreira, F. H. and Walton, M. (2005). World development report 2006: equity and development, volume 28. World Bank Publications.
- Fleurbaey, M. (1994). On fair compensation. Theory and decision, 36:277–307.

- Fleurbaey, M. (1995). Equality and responsibility. *European Economic Review*, 39(3-4):683–689.
- Fleurbaey, M. and Peragine, V. (2013). Ex ante versus ex post equality of opportunity. *Economica*, 80(317):118–130.
- Han, H. (2022). The utility of receiver operating characteristic curve in educational assessment: Performance prediction. *Mathematics*, 10(9):1493.
- Herrero, C., Iturbe-Ormaetxe, I., and Nieto, J. (1998). Ranking opportunity profiles on the basis of the common opportunities. *Mathematical Social Sciences*, 35(3):273–289.
- Hothorn, T., Hornik, K., and Zeileis, A. (2006). Unbiased recursive partitioning: A conditional inference framework. *Journal of Computational and Graphical statistics*, 15(3):651– 674.
- James, G., Witten, D., Hastie, T., Tibshirani, R., et al. (2013). An introduction to statistical learning, volume 112. Springer.
- Jaoul-Grammare, M. and Magdalou, B. (2013). Opportunities in higher education: An application to france. Annals of Economics and Statistics/Annales d'Économie et de Statistique, pages 295–325.
- Juárez, F. W. C. and Soloaga, I. (2014). iop: Estimating ex-ante inequality of opportunity. *The Stata Journal*, 14(4):830–846.
- Jusot, F., Tubeuf, S., and Trannoy, A. (2013). Circumstances and efforts: how important is their correlation for the measurement of inequality of opportunity in health? *Health economics*, 22(12):1470–1495.
- Kosor, M. M., Perovic, L. M., and Golem, S. (2019). Efficiency of public spending on higher education: a data envelopment analysis for eu-28. *Problems of Education in the 21st Century*, 77(3):396.
- Kranich, L. (1996). Equitable opportunities: an axiomatic approach. Journal of economic theory, 71(1):131–147.
- Lábaj, M., Luptáčik, M., and Nežinskỳ, E. (2014). Data envelopment analysis for measuring economic growth in terms of welfare beyond gdp. *Empirica*, 41:407–424.
- Marrero, G. A. and Rodríguez, J. G. (2012). Inequality of opportunity in europe. Review of Income and Wealth, 58(4):597–621.

- OECD (2024). Education at a glance 2024: Oecd indicators. OECD Publishing, Paris.
- Ok, E. A. (1997). On opportunity inequality measurement. *Journal of Economic Theory*, 77(2):300–329.
- Ok, E. A. and Kranich, L. (1998). The measurement of opportunity inequality: a cardinalitybased approach. *Social Choice and Welfare*, 15(2):263–287.
- Palmisano, F., Biagi, F., and Peragine, V. (2022). Inequality of opportunity in tertiary education: evidence from europe. *Research in Higher Education*, pages 1–52.
- Peragine, V. and Ferreira, F. (2015). Equality of opportunity: Theory and evidence. World Bank policy research paper, 7217.
- Peragine, V. and Serlenga, L. (2008). Higher education and equality of opportunity in italy. In *Inequality And Opportunity: Papers From The Second Ecineq Society Meeting*, pages 67–97. Emerald Group Publishing Limited.
- Perlich, C., Provost, F., and Simonoff, J. S. (2003). Tree induction vs. logistic regression: A learning-curve analysis. *Journal of Machine Learning Research*, 4(Jun):211–255.
- Roemer, J. E. (1993). A pragmatic theory of responsibility for the egalitarian planner. *Philosophy & Public Affairs*, pages 146–166.
- Roemer, J. E. (1998). Equality of opportunity. Harvard University Press.
- Roemer, J. E. (2006). Review essay," the 2006 world development report: Equity and development". *Journal of Economic Inequality*, 4(2):233.
- Roemer, J. E. and Trannoy, A. (2016). Equality of opportunity: Theory and measurement. Journal of Economic Literature, 54(4):1288–1332.
- Savaglio, E. and Vannucci, S. (2007). Filtral preorders and opportunity inequality. Journal of Economic Theory, 132(1):474–492.
- Sutherland, D. (2023). Public spending efficiency in compulsory education. In Handbook on Public Sector Efficiency, pages 251–273. Edward Elgar Publishing.
- Van De Gaer, D. (1995). Equality of opportunity and investment in human capital. *PhD* dissertation, Katholieke Universiteit Leuven.
- Weymark, J. A. (2003). Generalized gini indices of equality of opportunity. The Journal of Economic Inequality, 1:5–24.

A APPENDIX

A.1 Conditional inference tree

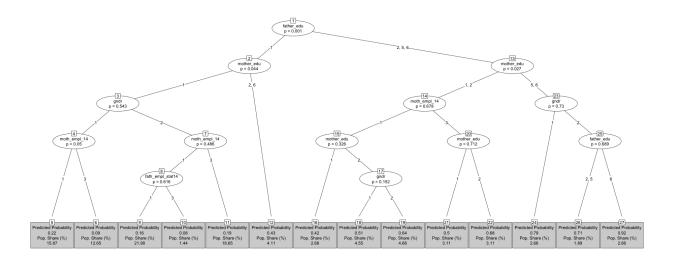


Figure 6: Conditional inference tree example - Portugal 2023

A.2 Unbalanced DEA results

Country	Input	Output	TE	Peer1	Peer2
BE	1.20	30.4	1.00	BE	NA
CZ	0.88	8.58	0.32	BE	IL
DE	1.02	14.4	0.50	BE	IL
\mathbf{EE}	1.04	23.8	0.83	BE	IL
ES	1.03	18.9	0.61	BE	IL
FI	1.75	21.6	0.66	NO	NA
\mathbf{FR}	1.18	12.41	0.41	BE	IL
IE	1.16	14.4	0.48	BE	IL
NL	1.12	22.41	0.75	BE	IL
NO	1.56	32.4	1.00	NO	NA
PL	1.02	20.28	0.71	BE	IL
CH	1.13	15.01	0.51	BE	IL
PT	0.91	7.92	0.29	BE	IL
SE	1.42	20.52	0.64	BE	IL
SI	1.021	14.04	0.49	BE	IL
DK	1.68	28.08	0.86	NO	NA
IL	0.82	26.36	1.00	IL	NA
LT	1.08	22.95	0.79	BE	IL
RU	0.97	23.6	0.74	BE	IL
SK	0.61	12.75	1.00	SK	NA
HU	0.81	12.24	0.48	$^{\mathrm{IL}}$	SK
Average	1.06	19.18	0.66		

Table 12: Technical efficiency and reference peers -2010

Country	Input	Output	TE	Peer1	Peer2
BE	1.26	34.44	0.99	NO	GB
CZ	0.90	13.14	0.42	NO	GB
DE	1.03	22.33	0.68	NO	GB
EE	1.12	28.7	0.86	NO	GB
ES	0.81	21.06	0.69	NO	GB
FI	1.39	30.42	0.85	NO	GB
\mathbf{FR}	1.12	18.96	0.57	NO	GB
IE	0.59	24.49	0.86	NO	GB
NL	1.13	32.37	0.97	NO	GB
NO	1.81	39.69	1.00	NO	NA
PL	0.94	21.56	0.68	NO	GB
CH	1.24	19.5	0.56	NO	GB
PT	0.67	20.3	0.69	NO	GB
SE	1.31	31.54	0.89	NO	GB
SI	0.87	19.5	0.63	NO	GB
AT	1.55	10.78	0.28	NO	GB
BG	0.67	17.94	0.61	NO	GB
DK	1.59	34.76	0.92	NO	GB
GB	0.49	27.39	1.00	GB	NA
HR	0.79	11.68	0.38	NO	GB
HU	0.72	11.84	0.40	NO	GB
IT	0.55	10.56	0.37	NO	GB
LT	0.71	19.44	0.66	NO	GB
SK	0.62	14.44	0.63	NO	GB
Average	0.88	23.88	0.63		

Table 13: Technical efficiency and reference peers – 2018

A.3 Production Possibilities Frontiers - DEA

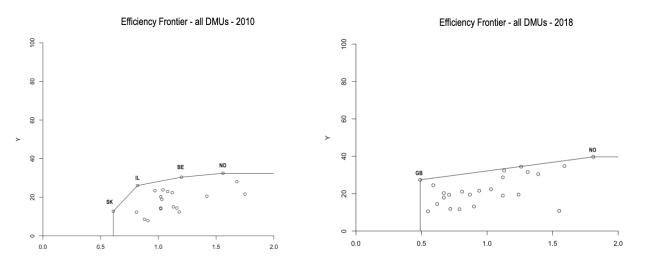


Figure 7: Production Possibilities Frontiers - 2010 and 2010 - all DMUs

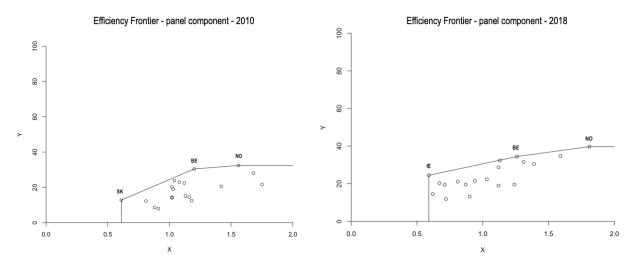


Figure 8: Production Possibilities Frontiers - 2010 and 2010 - Panel component