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Measuring inequality and welfare when some inequalities matter more than others

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Measuring inequality and welfare when some inequalities matter more than others*

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Abstract

This paper proposes a unified framework to measure inequality and social welfare in the case in which both inequalities between groups and inequalities within groups matter, but priority is recognized to the former. This novel approach can be applied to a variety of contexts, including the analysis of inequalities of opportunity, ethnic discrimination and gender disparities. The empirical part of the paper analyzes two relevant cases: (i) the evolution of income inequality and ethnic discrimination in the United States during the period 1970-2014; (ii) the comparison of four European countries - Italy, Spain, France and Germany - in terms of inequality of opportunity.

Keywords: Inequality; social welfare; horizontal inequalities; inequality of opportunity; ethnic discrimination.

JEL: D30, D63, I30.

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1 Introduction

The influential works of Piketty (Piketty & Saez, 2006; Piketty *et al.*, 2006; Alvaredo *et al.*, 2017; Piketty *et al.*, 2019) have brought income inequality at the center of public debates at the global level. In addition to general inequalities, there is often a special interest in disparities between groups, e.g., based on race or gender (esp. in sociology, e.g. Manduca, 2018), and one can attribute such a special interest to the widespread feeling that such inequalities are particularly unfair. The focus on "unfair inequalities" is also observed in the more recent literature on inequalities of opportunities (Ferreira & Peragine, 2016), which analyzes inequalities between groups of individuals sharing similar circumstances outside the individual responsibility, such as race and gender, but also parental background and similar sociodemographic characteristics. There is also an important literature on social mobility which looks at opportunities depending on parental ranking in the income distribution (Chetty *et al.*, 2014).

This literature on group disparities and on inequalities of opportunity has generally assumed that inequalities within the groups do not matter, and has often focused on the average income for each group, sometimes with a correction for heterogeneous sociodemographic characteristics.[1] Hence, one often finds summary measures of income inequality and, separately, different measures of group disparities and/or inequality of opportunity. However, in many context one is interested in the full picture of inequalities in a given society, but wants to recognize a special role, a priority, to inequality between well defined social groups. This issue appears particularly relevant for the study of inequalities of opportunities, because inequal-

[1] For example, if the age distribution is different in two groups, corrections to eliminate spurious results due to this fact can be implemented. See, for example, Lê Cook *et al.* (2010).

ities within groups may come in part from unobservable circumstances (such as genetic characteristics) that one would ideally want to include in the list of sources of unfair inequalities. In fact, some discontent has been expressed in the literature (see Kanbur & Wagstaff, 2016) with respect to the inequality of opportunity literature for its exclusive focus on inequality between circumstance-groups while ignoring (hence implicitly justifying) residual, sometimes very high, income inequalities. In this paper we address exactly this issue and propose a mixed measure in which inequalities within groups matter less than inequalities between groups, but are not completely dismissed as irrelevant. Hence in our approach both vertical income inequalities and horizontal inequalities do matter, but priority is recognized to the latter.

We therefore set out to study the construction of social welfare functions and of inequality measures that mix different degrees of inequality aversion between groups and within groups. The idea of such a mixture is related to recent contributions in the literature. Berger & Emmerling (2020) propose a variant of the Atkinson social welfare function that mimics nested CES production functions, and we will see this function as emerging from axiomatic analysis in our paper as well. Fleurbaey & Zuber (2023) study the transfer principles that could be associated to a greater aversion to inequality within groups than between groups (the opposite of what we investigate in this paper).[2] They mainly obtain negative results, although various social welfare functions appear in their analysis, including the generalized Atkinson functions also seen in Berger and Emmerling's work[3].

[2]The motivation for a greater aversion to inequality within groups applies, for instance, in the case in which groups are nations and solidarity is weaker across borders than within borders.

[3]A related contribution is contained in Hufe et al. (2022), in which the authors combine the equality of opportunity principle (hence between group inequality aversion) with the principle of poverty eradication

Our proposal to analyze hierarchical aversion to income inequality relies on a simple yet powerful normative principle which we can explain as follows. Suppose there are two groups, one of which is more advantaged than the other. A regressive transfer within the advantaged group may be mildly offensive as it increases the inequality within the group. However, a regressive transfer between groups (one in which the donor is in the disadvantaged group and is poorer than the recipient in the advantaged one) is more offensive, as it increases both inequality between groups and total inequality. Thanks to this requirement, together with other standard properties in the literature, we define a family of *between groups prioritarian* (BGP) social welfare functions. We then normalize social welfare in terms of equally distributed equivalent, so that it is immediate to derive *between groups prioritarian* (BGP) inequality indices *à la* Atkinson (1970).

We complete the theoretical framework by proposing a criterion, based on a generalization of the Lorenz curve, for robust partial rankings of distributions in terms of classes of BGP social welfare functions. Finally, we apply our proposal to the evaluation of the income distributions in two relevant contexts. The first one is an analysis of the income dynamics in the United States, when inequality between ethnic groups is considered particularly offensive. The second illustration compares four European countries - Italy, Spain, France and Germany - in terms of income inequality and prioritarian inequality of opportunity. Both illustration show that our proposal can lead to different comparisons of income distributions, and call for stronger redistributive policies which address a bigger share of the observed inequalities.

The paper is organised as follows. Section 2 outlines the theoretical framework, introduces the family of BGP social welfare functions, and describes the main transfer axiom characterizing it. A more formal exposition, together with the proofs of our main results can be found in the Appendix. Section 3 applies our proposal to analyse social welfare and inequality dynamics in the United states, and inequality of opportunity comparison across four european countries. Section 4 concludes.

2 Theoretical framework

2.1 Preliminary

We consider a population that can be partitioned in $n \in \mathbb{N}_{++}$ non-intersecting groups of dimension $m \in \mathbb{N}_{++}$. To shorten notation, let $\{1, ..., n\} = N$ and $\{1, ..., m\} = M$. In this setting, income distributions are represented as matrices $X \in \mathbb{R}^{n \times m}_{++}$, where each row $\mathbf{x}_j$ is the income distribution of a group $j \in N$, and x_{jk} is the income of an individual k in it. With a slight abuse of notation, we denote with $x_{j[r]}$ the income of the individual with rank r in the increasingly ordered permutation of $\mathbf{x}_j$.

Throughout the paper we rely on two key definitions. The first one formalizes the family of Schur-concave functions, which corresponds to the class of real-valued functions that are inequality-averse.

Definition 1. *For all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^p$, the function $f : \mathbb{R}^p \to \mathbb{R}$ is Schur-concave if and only if (i) $\sum_{i=1}^{k} x_{[i]} \geq \sum_{i=1}^{k} y_{[i]}$ for all $k = 1, ..., p$, and (ii) $\sum_{i=1}^{p} x_i = \sum_{i=1}^{p} y_i$, imply $f(\mathbf{x}) \geq f(\mathbf{y})$. We say that f is strictly Schur-concave if (i) and (ii) imply $f(\mathbf{x}) > f(\mathbf{y})$.*

Conditions (i) and (ii) in the above definition imply that $\mathbf{y}$ majorizes $\mathbf{x}$. The reader may refer to Marshall *et al.* (2011) for a book-length discussion of the relation between inequality, majorization and Schur-convexity.

The second definition is a method for comparing the degree of inequality aversion of two Schur-concave functions defined on different domains. Since the majorization test can be performed only on vectors of the same dimension, we rely on the Lorenz curve to establish whether two distributions of different size display the same degree of inequality (Marshall *et al.*, 2011).

Definition 2. *An increasingly monotone function $f : \mathbb{R}_+^p \to \mathbb{R}_+$ is more Schur-concave than an increasingly monotone function $g : \mathbb{R}_+^q \to \mathbb{R}_+$ if, for all $(\mathbf{x}, \mathbf{y}) \in \mathbb{R}_+^p \times \mathbb{R}_+^q$ such that $\mathbf{x}$ and $\mathbf{y}$ have the same mean and the same Lorenz curve*[4]*, one has $a \leq b$, where a, b are defined by $f(a, ..., a) = f(\mathbf{x})$ and $g(b, ..., b) = g(\mathbf{y})$.*

2.2 Social welfare functions

This section introduces a family of social welfare functions that allows us to rank income distributions by accounting for both inequality within and between the n groups. This class of social welfare functions is obtained axiomatically, by imposing a series of desirable properties. This section illustrates them in a simple and intuitive way; the interested reader may refer to Appendix A for the formal analysis and proofs.

We focus on the family of between-group prioritarian (BGP) social welfare func-

[4]For all increasingly ordered vectors $\mathbf{x} \in \mathbb{R}^n$, the Lorenz curve is the graph of the function $L(\mathbf{x}, k/n) = n^{-1} \left(\sum_{i=1}^{k} x_i / \sum_{i=1}^{n} x_i \right)$, $k = 1, ..., n$.

tions $W : \mathbb{R}^{n\times m}_{++} \rightarrow \mathbb{R}_+$ such that

$$W(X) = F\left(f\left(\mathbf{x}_1\right), \ldots, f\left(\mathbf{x}_n\right)\right) \tag{1}$$

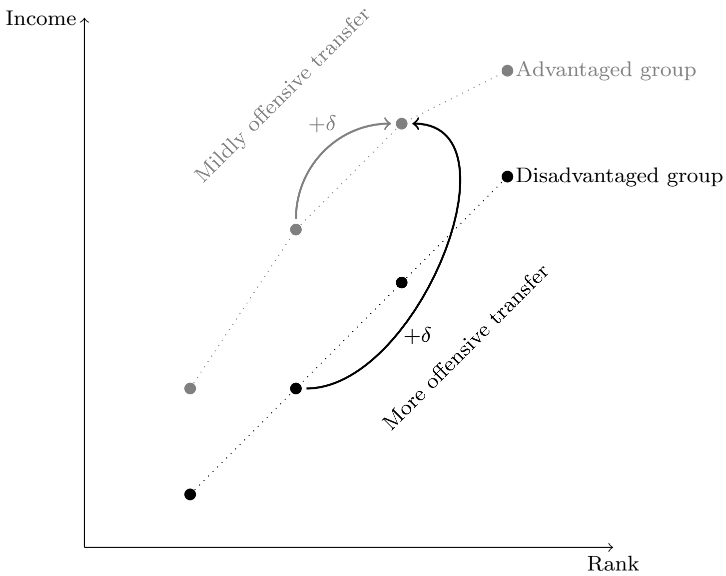
where $F : \mathbb{R}^n_+ \rightarrow \mathbb{R}_+$ and $f : \mathbb{R}^m_+ \rightarrow \mathbb{R}_+$ are both differentiable, strictly monotone, homogeneous of degree one, and such that $F(e, \ldots, e) = f(e, \ldots, e) = e$, f is Schur-concave, F is strictly Schur-concave and F is more Schur-concave than f.

Our social welfare functions are strictly increasing in income and equally distributed equivalent, so that if all individuals in the distribution X have the same income e, then $W(X) = e$. Eq. (1) starts by assessing the income distribution of each group with the inequality averse function f. The Schur-concavity of f captures the degree of aversion to the inequality within groups. The vector of equivalent incomes, that summarize the income distribution of each group, is then evaluated by the function F which is strictly Schur-concave.

Schur-concave functions are, by definition, symmetric. Therefore the contribution of each individual to the welfare of his group depends only on his income. Moreover, since the f function aggregating groups' income distributions is the same, groups are treated symmetrically and are *a priori* equally important.

The crucial feature of the BGP family is that F is more inequality averse than f, capturing the idea that social welfare is negatively impacted by inequality within groups but is more sensitive to inequality between groups. This feature of the SWFs reflects a basic transfer principle which can be easily described with the help of Figure 1. The graph displays the income parade of two groups. The advantaged group (gray dots) first order stochastically dominates the disadvantaged one (black dots). The gray arrow represents a regressive transfer from a poor to

Figure 1: Illustration of the main axiom on two income parades.



Description: The illustration displays the income parade of an advantaged (gray dots) and a disadvantaged (black dots) group. The two arrow represent regressive transfers occurring within a group (gray arrow) or between groups (black arrow). The latter transfer is more welfare reducing than the former.

a richer individual who belong to the same (advantaged) group. The black arrow represents a regressive transfer from a poor individual in the disadvantage group to richer individual in the advantaged group. Hence, both transfer increase inequality (they are regressive); in addition, the first transfer increases inequality within (the advantaged) group while the second transfer increases inequality between groups. Our Between-group Prioritarian Inequality Aversion Axiom declares that both transfer will reduce social welfare; however, the second transfer will reduce social welfare more than the first transfer. Formally, our main axiom is stated as follows.

For all $X, X', X'' \in \mathbb{R}_+^{n\times m}$, if there exist two groups $i, j \in N$ such that $x_{j[r]} \geq x_{i[r]}$ for all $r \in M$ and $\delta > 0$ such that $x_{j[k]} \geq x_{j[l]} \geq x_{j[l]} - \delta \geq x_{i[l]}$, and such that $x''_{j[k]} = x'_{j[k]} = \big(x_{j[k]} + \delta\big)$, $x'_{j[l]} = \big(x_{j[l]} - \delta\big)$, and $x''_{i[l]} = \big(x_{i[l]} - \delta\big)$, with X and X', and X and X'', coinciding everywhere else, then $W(X) \geq W(X') > W(X'')$.

In the Appendix A we show that the axiom of Prioritarian Inequality Aversion, when combined with other basic properties, is able to characterize the family of BGP social welfare functions introduced in Eq. (1). The characterization result is particularly interesting from a theoretical perspective, and the family of social welfare functions that can be written as Eq. (1) results to be quite rich. Moreover, by imposing additivity of both F and f, we obtain a subclass of Eq. (1) in which the degrees of aversion to inequality within and between groups appear as explicit parameters. Formally, we have

$$W_{\beta,\omega}(X) = \left[\frac{1}{n}\sum_{j=1}^{n}\left(\frac{1}{m}\sum_{k=1}^{m} x_{jk}^{1-\omega}\right)^{\frac{1-\beta}{1-\omega}}\right]^{\frac{1}{1-\beta}} \tag{2}$$

where $\beta \geq \omega \geq 0$. In words, Eq. (2) is the generalized mean of order $(1-\beta)$ of the generalized means of order $(1-\omega)$ of the income distributions of each group. The parameter $\omega \geq 0$ measures the degree of within group inequality aversion. The higher is ω, the more sensitive is W to within groups inequality, with $\omega = 0$ corresponding to the case of no inequality aversion. Similarly, the parameter β measures aversion to between groups inequality, with higher values of the parameter being associated with stronger aversion. The reader may notice the following special cases: (1) if $\beta = \omega = 0$, then social welfare corresponds to the average income; (2) if $\beta = \omega > 0$, then within and between groups inequality are equally relevant, and social welfare is simply a generalized mean of the income distribution; (3) if $\beta > 0$ and $\omega = 0$, then the only inequality that is detrimental for social welfare is the one between groups.

A relevant advantage of an equally distributed equivalent social welfare function is its one to one relation with an Atkinson type of inequality index. Formally, let $I : \mathbb{R}_{++}^{n\times m} \to \mathbb{R}$ be such that

$$I(X) = 1 - \frac{W(X)}{\mu(X)} \tag{3}$$

for some W as in Eq. 1, where $\mu(X)$ is the average income in X. Then, I is a BGP inequality index for which between groups inequality matters more than within groups inequality.

Particularly: (i) I is continuous; (ii) $I(X) = 0$ if X shows between and within group equality; (iii) $I(X) \leq I(Y)$ if X can be obtained from Y through a series of welfare increasing transfers. Since social welfare is maximized when income is equally distributed, this index measures the fraction of the total income that a social planner is willing to sacrifice, in order to remove income inequality. This fraction will be higher when a bigger share of the total inequality is occurring between groups.

When BGP social welfare takes the parametric form in Eq. (2), we have

$$I_{\beta,\omega}(X) = 1 - \left[\frac{1}{n} \sum_{j=1}^{n} \left(\frac{1}{m} \sum_{k=1}^{m} \left(\frac{x_{jk}}{\mu(X)} \right)^{1-\omega} \right)^{\frac{1-\beta}{1-\omega}} \right]^{\frac{1}{1-\beta}} \tag{4}$$

which is a BGP inequality index in which $\beta \geq 0$ is the parameter that defines our aversion to between groups inequality, and $0 \leq \omega \leq \beta$ expresses the degree of aversion to inequality within groups.

We conclude this section with a comment on the income distributions' domain. We assumed, for simplicity, that all groups have the same size. Blackorby *et al.* (2001) show that under a mild strengthening of continuity, we can define equally distributed equivalent measures for variable population size. In our setting this would amount to defining a series of functions $f^t : \mathbb{R}^t_+ \rightarrow \mathbb{R}_+$, one for each possible group's size $t \in \mathbb{N}$, and a value function $V : \mathbb{N} \times \mathbb{R} \rightarrow \mathbb{R}$ such that $V(m_j, f^{m_j}(\mathbf{x}_j))$ evaluates the distribution of a group j of size m_j (see also Zoli *et al.*, 2009, for a deeper discussion). These value functions should replace the relative $f(\mathbf{x}_j)$ in Eq. 1. Alternatively, one can deal with different population sizes by simply transforming the empirical distribution of a group j, say $\mathbf{y}_j = (y_{j1}, y_{j2}, ..., y_{jq})$ into an equivalent distribution $\mathbf{x}_j = (x_{j1}, x_{j2}, ..., x_{jm})$ with the same average and the same cumulative distribution function.

2.3 Robust comparisons

The previous section defined families of social welfare functions. Each instance of Eq. (1) or (2) generates a complete order of income distributions. It is well-known, however, that different social welfare functions may disagree on the ranking of two distributions, say X and Y. It is therefore useful to identify conditions that allow us to establish whether $W(X) \geq W(Y)$ holds independently of the chosen function.

For all $X \in \mathbb{R}^{n\times m}_{++}$ let $L(X)$ and $gL(X) = \mu(X)L(X)$ denote, respectively, the Lorenz and the generalized Lorenz curves of X. Moreover, for any vector, $\mathbf{z} \in \mathbb{R}^m_+$, let $\mu_\rho(\mathbf{z}) = \left(\frac{1}{m}\sum_{k=1}^m z_k^\rho\right)^{1/\rho}$ denote its mean of order ρ. Then, we can define the ρ-smoothed distribution X_{μ_ρ} which is the smoothed distribution obtained by substituting each element of X with the order ρ mean of the row it belongs to. Clearly, $\mu_1(\mathbf{z}) \equiv \mu(\mathbf{z})$ and $X_\mu \equiv X_{\mu_1}$.

Peragine (2004) shows that, for all $X, Y \in \mathbb{R}^{n\times m}_{++}$, $gL(X_\mu) \geq gL(Y_\mu)$ is equivalent to $W_{\beta,0}(X) \geq W_{\beta,0}(Y)$, for all $\beta \geq 0$. Moreover, if F and f are generalized means of the same order $1-\beta$ (so that they are equally Schur-concave) then, by Shorrocks (1983), $gL(X) \geq gL(Y)$ if and only if $W(X)_{\beta,\beta} \geq W_{\beta,\beta}(Y)$, for all $\beta \geq 0$.

The previous are, however, special cases of BGP social welfare functions, that do not extend to the wider family in Eq. (1). We can nevertheless identify sufficient conditions to ensure stability of the ranking between two income distributions. Formally, let X and Y be two distribution whose rows are named such that $\mu(\mathbf{x}_1) \leq ... \leq \mu(\mathbf{x}_n)$ and $\mu(\mathbf{y}_1) \leq ... \leq \mu(\mathbf{y}_n)$.[5] If $L(\mathbf{x}_j) \geq L(\mathbf{y}_j)$ for all $j = 1, ..., n$, and $gL(X_\mu) \geq gL(Y_\mu)$, then $W(X) \geq W(Y)$ for all W as in Eq. (1).

The reader may refer to Appendix B for a proof of the above result. We should however notice how this test identifies sufficient but not necessary conditions. Indeed, $L(\mathbf{x}_j) \geq L(\mathbf{y}_j)$ for all $j = 1, ..., n$ ensures $W(X) \geq W(Y)$ even when within group inequality aversion is higher than the between groups one. The difficulty of obtaining a clean alternative to this dominance condition comes from our assumption that groups should have the same importance for social welfare. [6]

The previous test leaves us unsatisfied because it does not really capture our priority to between-group inequality. An interesting dominance condition is however obtained if we impose a minimum level of aversion to within groups inequality, and between group inequality aversion strictly higher. In other word, let us focus on the set of $W_{\beta,\omega}$ functions such that $\omega = \bar{\omega} \geq 0$ and $\beta \geq \bar{\omega} + \epsilon$, for some small $\epsilon > 0$. As also shown in Meyer (1975, 1977)'s result about stochastic dominance

[5] The reader may notice that, since groups are treated symmetrically by our social welfare functions, any distribution Z can be row-permuted without altering $W(Z)$.

[6] In a different framework, assuming a pre-order of groups and giving priority to worse off groups, Eq. 2 would become a "need-based" welfare function. For this class of functions, Ok & Lambert (1999) show that *sequential* generalized Lorenz dominance is then the only test that characterizes partial robust rankings.

with respect to a function,[7] for all $X, Y \in \mathbb{R}^{n\times m}_{++}$, $gL\left(X_{\mu_{(1-\bar{\omega})}}\right) \geq gL\left(Y_{\mu_{(1-\bar{\omega})}}\right)$ if and only if $W_{\beta,\bar{\omega}}\left(X\right) \geq W_{\beta,\bar{\omega}}\left(Y\right)$ for all $\beta \geq \bar{\omega} + \epsilon$. In other words, once we fix the degree of aversion to inequality within groups (ω), and we focus on the family of additive BGP social welfare functions, we can easily check if the ranking of X and Y is robust to different degrees of between groups inequality aversion (β). This generalized Lorenz dominance test is interesting and simple to implement. Moreover, it underlines the higher relevance of between groups inequality, because it compares the two distributions for all admissible values of β.

We conclude this section with a numerical example of the two tests described in this section. Let us consider the following distributions, whose rows are all ordered by their mean.

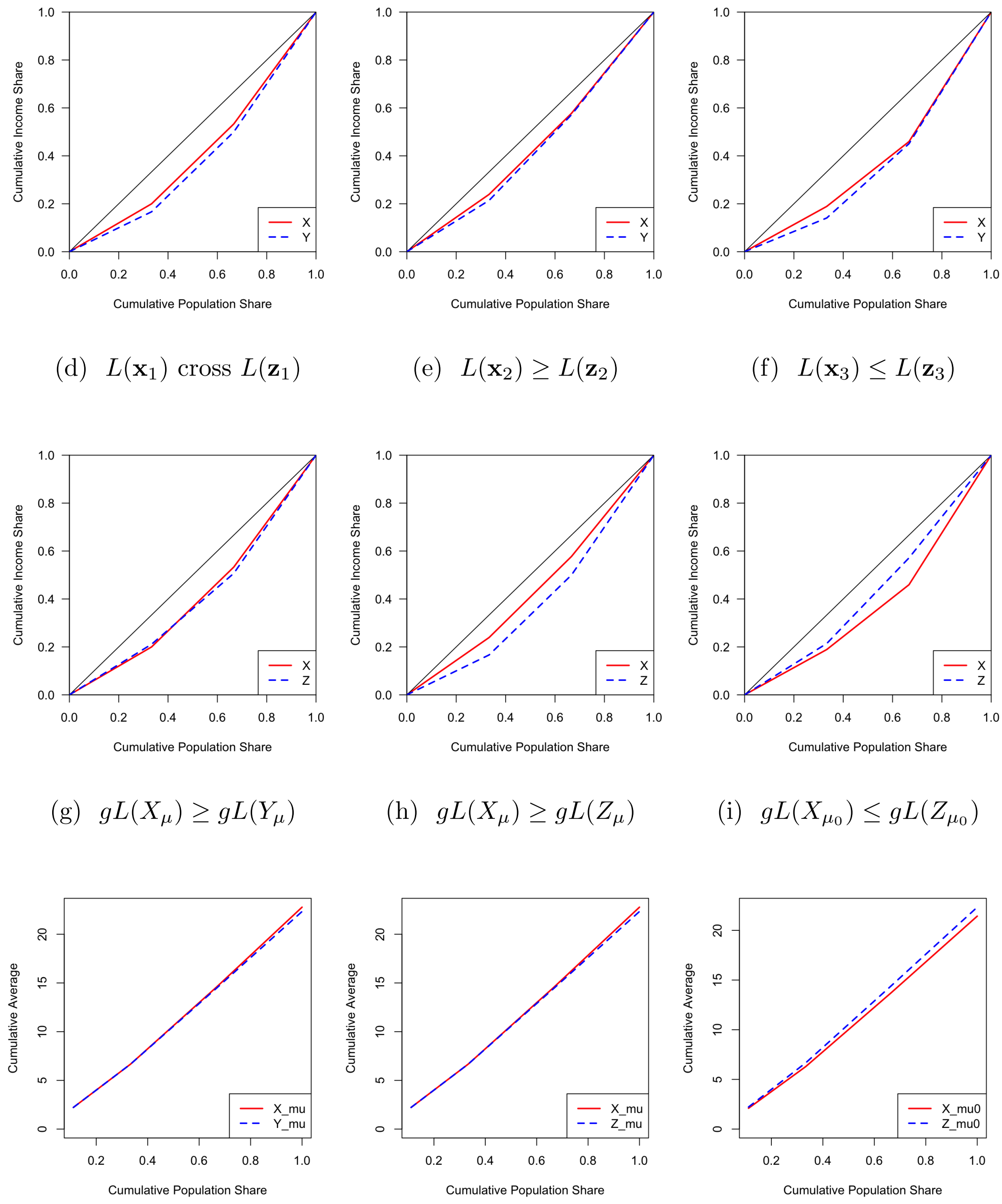
$$X = \begin{bmatrix} 12 & 20 & 28 \\ 17 & 24 & 30 \\ 14 & 20 & 40 \end{bmatrix} Y = \begin{bmatrix} 10 & 20 & 30 \\ 15 & 25 & 30 \\ 10 & 22 & 39 \end{bmatrix} Z = \begin{bmatrix} 1.5 & 2.1 & 3.5 \\ 10 & 20 & 30 \\ 15 & 25 & 30 \end{bmatrix}$$

It is easy to see (Figure 2) that $L\left(\mathbf{x}_j\right) \geq L\left(\mathbf{y}_j\right)$ for all $j = 1, 2, 3$. Moreover, $gL(X_\mu) \geq gL(Y_\mu)$. Therefore, we can conclude that $W(X) \geq W(Y)$ for all W as in Eq. 1.

The same cannot be said for X and Z. In particular, $L\left(\mathbf{x}_1\right)$ and $L\left(\mathbf{z}_1\right)$ cross, while $L\left(\mathbf{x}_2\right) \geq L\left(\mathbf{z}_2\right)$ and $L\left(\mathbf{x}_3\right) \leq L\left(\mathbf{z}_3\right)$. Nevertheless, $gL(X_\mu) \geq gL(Z_\mu)$, so that if one expresses zero aversion to within groups inequality, then $W(X) \geq W(Z)$ for all admissible BGP social welfare functions. This, however, depends on the degree of aversion to within groups inequality. To see this, observe that $gL(X_{\mu_0}) \geq gL(Z_{\mu_0})$, so that we can apply Meyer (1975, 1977)'s result to conclude

[7] See also Zheng (2022) on the link between this and the generalized Lorenz dominance.

Figure 2: Illustration dominance conditions



(a) $L(\mathbf{x}_1) \geq L(\mathbf{y}_1)$ (b) $L(\mathbf{x}_2) \geq L(\mathbf{y}_2)$ (c) $L(\mathbf{x}_3) \geq L(\mathbf{y}_3)$

(d) $L(\mathbf{x}_1)$ cross $L(\mathbf{z}_1)$ (e) $L(\mathbf{x}_2) \geq L(\mathbf{z}_2)$ (f) $L(\mathbf{x}_3) \leq L(\mathbf{z}_3)$

(g) $gL(X_\mu) \geq gL(Y_\mu)$ (h) $gL(X_\mu) \geq gL(Z_\mu)$ (i) $gL(X_{\mu_0}) \leq gL(Z_{\mu_0})$

that $W_{\beta,1}(X) \geq W_{\beta,1}(Z)$ for all $\beta > 1$.

3 Application

The theoretical proposal explained in the previous sections can be applied to a variety of settings. In this section we propose two empirical analyses: the first one looks at the United States and illustrate the case in which, when evaluating income dynamics, in addition to aversion to income inequality there is a special (stronger) concern for inequality between ethnic groups; the second illustration looks at inequality of opportunity in four European countries, showing the consequences of introducing aversion to inequality of income together with a stronger aversion to inequality of opportunity.

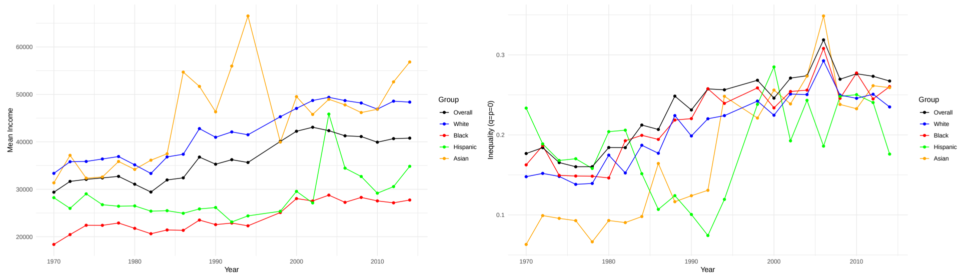
3.1 Ethnic discrimination and income inequality in the United States

The literature is rich of works analysing ethnic discrimination and income inequalities between ethnic groups in the United States (see, for example, Darity *et al.*, 1997; Darity Jr & Nembhard, 2000; Akee *et al.*, 2019). These are only part of the many studies that look at the evolution of income inequality in this country (Piketty & Saez, 2003; Auten & Splinter, 2024, among others). In this empirical analysis we merge the concern for inequality and ethnic discrimination within a unique framework that recognizes priority to inequalities between ethnic groups.

We explore the evolution of income inequality in the United States using data from the Panel Study of Income Dynamics (PSID) between 1970 and 2014. PSID

Figure 3: Trends in mean income and inequality in the United States

(a) Per-capita income (b) Inequality



Description: The graphs show the per-capita income (panel a) and inequality (panel b) trends. Both measure are computed for the overall population and for each ethnic group: White, Black, Hispanic and Asian. Inequality is measured as $1 - (W_{1,1}/W_{0,0})$.

Source: Own elaboration based on PSID data.

is a panel study covering more than 18,000 individuals from about 5,000 households in the US. It provides nationally representative information on the US based non-immigrant population. Our outcome of interest is the equivalized disposable household income in dollars PPP 2017.[8] We focus on the inequality between individuals belonging to different ethnic groups: White, Black, Hispanic, Asian (or Pacific islander), American Indian (or Aleut or Eskimo), and Others. To simplify the graphical analysis, we do not report the results for the last two groups, which are however considered in the computations.

Figure 3(a) shows that, between 1970 and 2014, the average income of Black and White Americans has been rising, with few exceptions in correspondence of the economic recessions. The Asian and Hispanic groups experienced greater volatility in average income growth but they also set on a positive trend. Moreover, in 2014, White and Asian result to be the two richest groups, while Black remains the poorest one for the entire period. Interestingly, the group's average income trends

[8]The normalization is done using the square root scale.

of White and Black mimic the aggregate one.

As shown by panel (b) of Figure 3, income growth has been accompanied by an increase in income inequality both in the aggregate and within each group. Income inequality is measured as $1 - (W_{1,1}(X)/W_{0,0}(X))$, which corresponds to the Atkinson inequality index with inequality aversion parameter equal to one. The trend is, once again, stable for the group of White and Black, while Asian and Hispanic show greater instability. This figure shows another interesting dynamic: over time, the inequality within the group of White has been evolving in the same way as the overall one, while the other groups display very different dynamics. At the same time, we observe a convergence of the inequality within each group toward the total inequality. Overall, during the 44 years we are considering, individuals within ethnic groups are becoming more heterogeneous. This justifies our concern for within groups inequality.

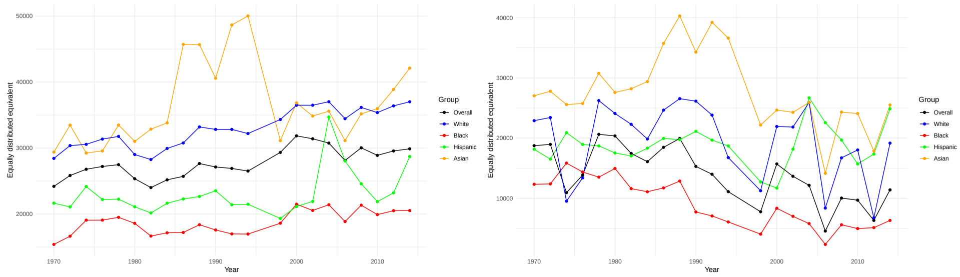
To better appreciate how increasing inequality affects the social assessment of ethnic groups, Figure 4(a) shows the evolution of their equally distributed equivalent income. It is immediate to see how the positive trends in average income are flattened when we account for inequality. Interestingly, these trends become even negative when we increase aversion to inequality; this is the case of Figure 4(b), where the inequality aversion parameter passes from 1 to 2.[9]

Figure 5(a) displays the evolution of the per-capita income, together with three social welfare functions obtained by changing the parameters β and ω in Eq. (2). A social welfare function that is averse to total income inequality, without making any distinction on the type of inequality, is obtained by setting $\beta = \omega = 1$ in

[9]The equally distributed equivalent is the mean or order $1 - \rho$ of a distribution. Figure 4(a) considers the geometric mean (mean of order 0) while panel (b) uses the harmonic one (mean of order -1).

Figure 4: Trends in equally distributed equivalent income in the United States

(a) $\omega = 1$ (b) $\omega = 2$



Description: The graphs show the evolution of the generalized average per-capita income of the population and of the White, Black, Hispanic and Asian groups. Panel (a) computes generalized means of order 0 (geometric means), while panel (b) shows generalized means of order -1 (harmonic means).

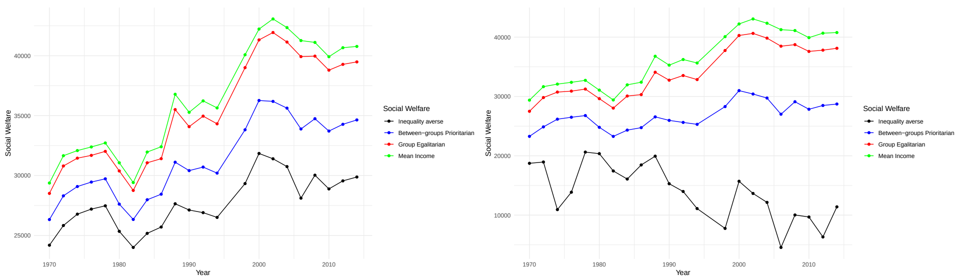
Source: Own elaboration based on PSID data.

Eq. (2). A social welfare function that is averse only to inequality between ethnic groups, is obtained by setting $\omega = 0$ and $\beta = 1$. Finally, a BGP social welfare function that is averse to income inequality but gives priority to the inequality between ethnic groups is obtained by setting $\omega = 0.5$ and $\beta = 1$.

The four curves in Figure 5(a) display a similar trend, with levels that become lower when more inequalities are considered welfare reducing. We may observe how the vertical distance between the mean income and the other three lines is increasing over time. This is the result of the rising inequality observed in Figure 3.(b). It is interesting to notice, that our between-group prioritarian social welfare (the blue line in Figure 5(a)) can lead to different assessments of the social welfare evolution. For example, while average income and group egalitarian social welfare increase from 2000 to 2002, our BGP social welfare decreases. Another example is the period 1990-1992 where inequality averse social welfare decreases, contrary to the BGP one.

Figure 5: Trends in Social Welfare in the United States

(a) (b)



Description: The graphs show the evolution of four measures of social welfare: (1) the average income ($W_{0,0}$), (2) the inequality averse social welfare ($W_{\beta,\beta}$), (3) the opportunity egalitarian social welfare ($W_{\beta,0}$), and (4) the between-group prioritarian social welfare ($W_{\beta,\omega}$). In panel (a) we set $\beta = 1$ and $\omega = 0.5$, while in panel (b) $\beta = 2$ and $\omega = 1$.

Source: Own elaboration based on PSID data.

Figure 5(b) replicates the analysis by imposing higher aversion to inequality. In particular, we set $\omega = \beta = 2$ for inequality averse social welfare, $\omega = 0$ and $\beta = 2$ for the group egalitarian one, and $\omega = 1$ and $\beta = 2$ for our BGP social welfare. Once again, the blue line shows similar trends with respect to the green and red ones, but we can notice some contradictions like in the period 1990-1992. This figure underlines how stronger concern for inequalities impacts our perception of social welfare in the US. The black line in Figure 5(b) denotes a negative social welfare trend, while the other curves, and in particular the one for our BGP social welfare, result much flatter.

The reader may notice how the red and black curves in Figure 5 constitute the upper and lower bounds for BGP social welfare functions with between group inequality aversion parameter $\beta = 1$ (panel a) or $\beta = 2$ (panel b). Within our setting, expressing stronger aversion to inequality between groups (increasing β) enlarges the admissible values for ω, and the distance between the upper and lower bounds of our BGP social welfare. Nevertheless, the bigger distance between the red and black curves in 2014 with respect to 1970 is driven by the rising inequality within ethnic groups. Figures 5(a) and (b) are clear evidence of how important this inequality has now become, and the relevance of a BGP approach to analyse social welfare dynamics.

3.2 Prioritarian inequality of opportunity in four European countries

The equality of opportunity (EOp) paradigm, developed after the seminal contributions of Sen (1980), Dworkin (1981a,b) and several political philosophers afterwards, puts particular emphasis on the remuneration of individual's effort. The EOp approach in fact defines fairness as the combination of two principles of social justice. The first one - the principle of compensation - aims at removing the inequalities caused by circumstances out of individual control. The second one - the reward principle - limits the scope of redistribution by addressing the problem of how to apportion the final outcome to effort. Different versions of compensation and reward have been proposed in the literature. By far, the most famous version of reward used, often implicitly, in empirical measures of inequality of opportunity is the so called "utilitarian reward", stating that all outcome inequality stemming from the exercise of effort is fair and does not call for compensation. More precisely, the version of EOp based on utilitarian reward combines aversion to inequalities due to circumstances (compensation) and complete neutrality with respect to inequalities due to effort (reward).

The between groups prioritarianism proposed in this paper can be seen as challenging utilitarian reward by suggesting that not all inequalities due to effort are fair. Hence, within the group of individuals with the same circumstances - a *type*, using the classic terminology - there is scope for additional fairness principles to limit inequalities. Our proposal addresses some of the critiques moved to the utilitarian reward, which may go against higher normative principles of fairness (see Fleurbaey, 1995, for example).

We explore inequality of opportunity (IOp) in Italy, Germany, France, and Spain using the European Union Statistics on Income and Living Conditions (EU-SILC) data for 2005, 2011 and 2019.

Our outcomes of interest are household incomes equivalized to account for the household size and normalize to USD 2010. We consider gender, mother and father occupation, mother and father education, and place of birth as circumstances out of individual control. Within each country-year the sample is partitioned into types (groups of individuals with the same circumstances) via Conditional Inference Trees: a data-driven methodology proposed by Hothorn *et al.* (2006) and often applied in the literature (see, for example, Brunori *et al.*, 2023).[10]

Figure 6(a) compares income inequality - measured as $1 - (W_{1,1}(X) / W_{0,0}(X))$ - and absolute IOp - measured as $1 - (W_{1,0}(X) / W_{0,0}(X))$ - for each country and year. In Figure 6(b) we refer to the concept of relative IOp, which is the ratio between absolute IOp and total inequality. It is well known in the literature that inequality and IOp are positively correlated. This is also evident in our setting. Nevertheless, as we can see from the red dots (2005) in panel (a), in smaller set of countries one can observe a negative correlation. More in general, if we focus on

[10] We use the *partykit* package written by Hothorn & Zeileis (2015) in combination with a tuning process to identify the model that best predicts individual expected income based on circumstances.

Figure 6: Comparing income and opportunity inequality.

(a) Absolute IOp (b) Relative IOp

Description: The two graphs plot income inequality - measured as $1 - (W_{1,1}(X) / W_{0,0}(X))$ - against absolute IOp - measured as $1 - (W_{1,0}(X) / W_{0,0}(X))$ - in panel (a) or relative IOp which is the ration between absolute IOp and income inequality. Country-year observations are pooled but represented with different colors.

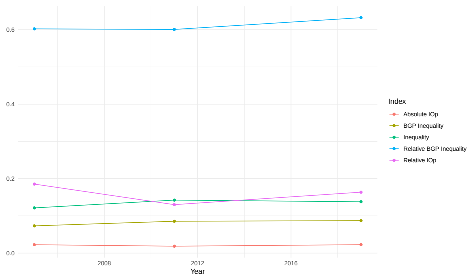
Source: Own elaboration based on EU-SILC data.

pairwise country comparisons, then it is evident that higher inequality does not necessary imply higher IOp. See, for example, Germany and Spain in 2005, or France and Italy in 2019. These are particular examples of country comparisons that are likely to depend on the degree of aversion to inequality within types. We should however notice that in the year 2011 we observe a strong correlation between IOp and income inequality. As we will see, this will make our country comparisons less sensitive to the introduction of a BGP approach.

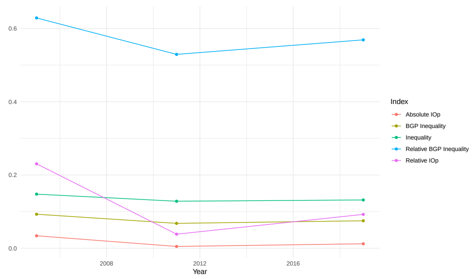
Before passing to the country comparisons, let us explore how inequality of opportunity and BGP inequality changed during the considered periods. Formally, BDP inequality is measured as as $1 - (W_{1,0.5}(X) / W_{0,0}(X))$. In other words, our BGP inequality index corresponds to the above IOp measure, except for the within type inequality aversion parameter (ω) that is set to 0.5: the middle point between aversion to only between group inequality ($\omega = 0$) and aversion to all income

Figure 7: Trends in inequality measures

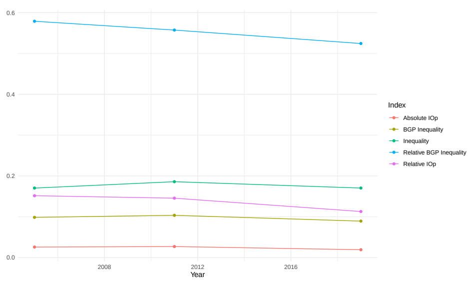
(a) France



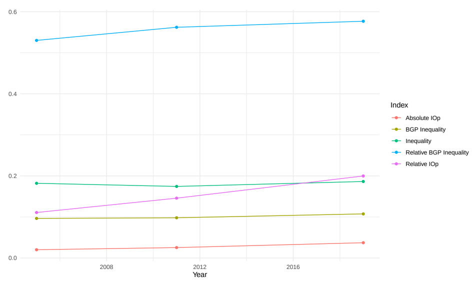
(b) Germany



(c) Italy



(d) Spain



Description: The graphs display the evolution of IOp and BGP inequality expressed both in absolute and relative terms (than is, as share of total income inequality). The indices are obtained as $1 - (W_{1,\omega}(X)/W_{0,0}(X))$, for $\omega = 0$ in the case of IOp and $\omega = 0.5$ in the case of BGP inequality.

Source: Own elaborations based on EU-SILC data.

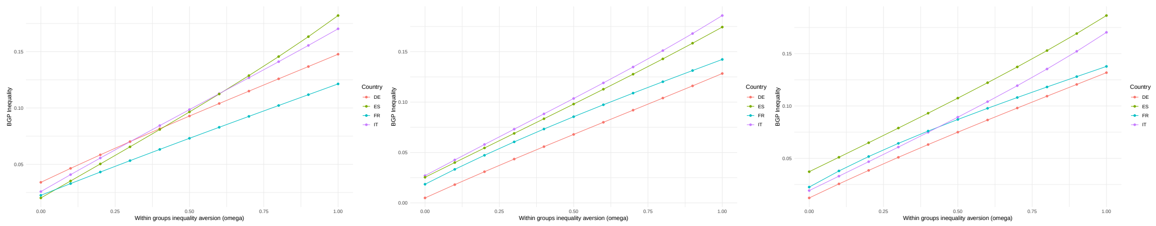
inequalities$(\omega = \beta = 1)$.

In Figure 7 we report both absolute and relative values of the two indices. The differences in levels between inequality, absolute IOp and absolute BGP inequality behaves as expected. It is however interesting to observe how big it is the difference in levels between relative IOp and relative BGP inequality. The former can be seen as the share of total income inequality that is condemned by an opportunity egalitarian society. This share is rarely above 20%. A society in line with our proposal, instead, condemns more than 50% of total income inequality.

For a deeper understanding of the role played by within type inequality aversion,

Figure 8: Sensitivity to within type inequality aversion.

(a) 2005 (b) 2011 (c) 2019



Description: The graphs display how BGP inequality in the four countries varies for different levels of aversion to within groups inequality. We do so by letting the parameter ω vary from neutrality to within types inequality ($\omega = 0$), to complete aversion ($\omega = \beta = 1$).

Source: Own elaborations based on EU-SILC data.

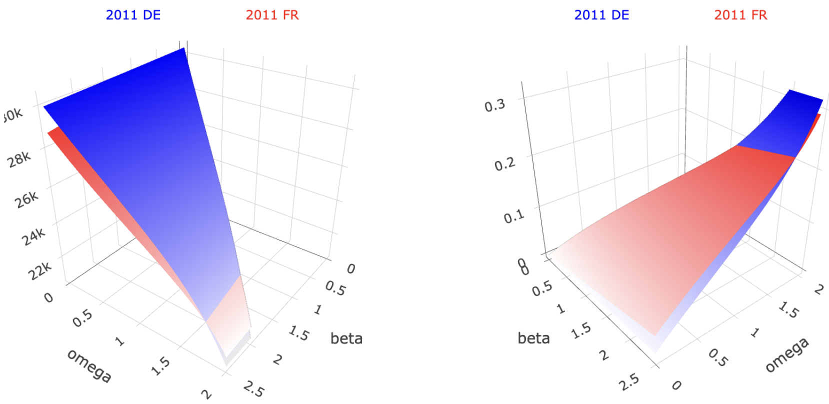
we investigate the sensitivity of the countries' ranking to changes in the parameter ω from complete aversion to within types inequality ($\omega = \beta = 1$), to neutrality ($\omega = 0$). Clearly, any intermediate position ($\omega \in (0, 1)$) is a different instance of a BGP inequality index.

Figure 8(a) is probably the most interesting evidence of how the BGP approach can influence country comparisons. Spain results to be the most unequal of the four countries, but also the one that displays lower between types inequality. It is immediate to see here how different degrees of within type inequality aversion lead to different country comparisons. Some sensitivity is also observed in 2019 (Figure 8(c)) where France and Italy rank differently depending on ω. Figure 8(b), instead, confirms our previous observation concerning the year 2011. The high covariance between IOp and income inequality makes the country comparison less sensitive to the BGP approach.

Our proposal is however not irrelevant for the year 2011. Indeed, if we enlarge the spectrum of possible values for β and ω, we see that the BGP approach may

Figure 9: Sensitivity to within and between type inequality aversion. A comparison between France and Germany in 2011.

(a) Social Welfare (b) Inequality



Description: The graphs compare BGP social welfare - measured as $W_{\beta,\omega}$- and inequality - measured as $1 - W_{\beta,\omega}/W_{0,0}$ - for various combinations of the parameters β and ω, expressing, respectively, aversion to between and within types inequality.

Source: Own elaboration based on EU-SILC data.

change the way we rank Germany and France. This is shown in Figures 9(a) and (b), where we graph, respectively, BGP social welfare and BGP inequality for the two countries, with different combinations of β and ω. As we can see, for a sufficiently strong aversion to within type inequality ($\omega > 1.5$) France performs better than Germany in terms of both BGP social welfare and inequality.

Overall, this shows that aversion to within type inequality does play a role in ranking countries in terms of IOp, and that our principle of (weak) aversion to within type inequality is relevant both from a normative and empirical perspective.

4 Concluding remarks

In this paper we have proposed a mixed measure of inequality and social welfare in which inequalities within groups matter less than inequalities between groups, but are not completely dismissed as irrelevant. In our approach both vertical income inequalities and horizontal inequalities do matter, but priority is recognized to the latter. This framework can be applied to a variety of contexts, including, for example, the analysis of inequalities of opportunity, ethnic inequalities and gender disparities. We have applied it to two relevant cases. The first one is an analysis of the income dynamics in the United States, when inequality between ethnic groups is considered particularly offensive. The second application compares four European countries - Italy, Spain, France and Germany - in terms of income inequality and prioritarian inequality of opportunity. Both illustration show that our proposal can lead to different comparisons of income distributions, and call for stronger redistributive policies which address a bigger share of the observed inequalities.

Appendix

A Axiomatic analysis

We assume that there exists a continuous ordering $\succsim$ that defines the preferences of society. By a well known result in the economic literature, $\succsim$ can be represented by a continuous social welfare function $W : D \to \mathbb{R}_+$, where $D = \mathbb{R}^{n\times m}_{++}$. We assume W to be also differentiable and characterize it imposing the following requirements. The first axiom says that an increase in one individual's income, everything else equal, improves social welfare.

Axiom 1. *Monotonicity (MON) - For all $X, X' \in D$, if there exist $(j,k) \in N \times M$ such that $x_{jk} > x'_{jk}$, with X and X' coinciding everywhere else, then $W(X) > W(X')$.*

The second axiom normalizes W to make it an equally-distributed equivalent so that when the total income is equally distributed in the population, the social welfare corresponds to the average income.

Axiom 2. *Normalization (NORM) - For all $X \in D$, if $x_{ik} = x_{jl} = e$ for all $(i,k),(j,l) \in N \times M$, then $W(X) = e$.*

Let us indicate with $X_{-j} \in \mathbb{R}^{(n-1)\times m}_{+}$ the matrix generated form X by dropping the j-th row. The next axiom imposes each group to be assessed independently of the others, and the comparison between two groups to be scale invariant.

Axiom 3. *Homothetic independence (IND) - For all $X, X' \in D$, $\lambda \geq 0$ and $j \in N$, $W(\mathbf{x}_j, X_{-j}) \geq W(\mathbf{x}'_j, X_{-j})$ if and only if $W(\lambda\mathbf{x}_j, X'_{-j}) \geq W(\lambda\mathbf{x}'_j, X'_{-j})$.*

The next axiom completes the previous one by stating that the only important information to assess groups is their incomes, so that changing the name or the position of a group in the income distribution should have no effect on the social welfare.

Axiom 4. *Group symmetry (SYM) -For all $X, X' \in D$, if there exist $i, j \in N$ such that $\mathbf{x}_i = \mathbf{x'}_j$ and $\mathbf{x}_j = \mathbf{x'}_i$, with X and X' identical otherwise, then $W(X) = W(X')$.*

The fifth axiom applies the symmetry property within each group, claiming that the contribution of an individual to the welfare of his group should depend only on his income, not on his name or his position in the group's distribution. Therefore, changing the position of individuals within a group should not affect the social welfare.

Axiom 5. *Within group symmetry (WSYM) -For all $X, X' \in D$, if there exist $(j,k),(j,h) \in N \times M$ such that $x_{jk} = x'_{jh}$ and $x_{jh} = x'_{jk}$, with X and X' identical otherwise, then $W(X) = W(X')$.*

The last axiom constitutes the main requirement characterizing the preferences of our society. It says that when a regressive transfer has to take place, it is better if it is within a group than between two groups, where one is unambiguously worse than the other. Let us consider an example where $(x_{11}, x_{12}) > (x_{21}, x_{22})$, $x_{12} \geq x_{11}$, $x_{22} \geq x_{21}$, $x_{11} - \delta \geq x_{21}$, $\delta > 0$ and

$$X = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}; X' = \begin{bmatrix} x_{11} - \delta & x_{12} + \delta \\ x_{21} & x_{22} \end{bmatrix}; X'' = \begin{bmatrix} x_{11} & x_{12} + \delta \\ x_{21} - \delta & x_{22} \end{bmatrix}$$

Since incomes in group 1 are pairwise higher than incomes in group 2, any appealing group's order should conclude that the former is better off. X'' is obtained from X via a transfer that worsens the situation of the worse off group, and X' is obtained from X after a regressive transfer within the advantaged group. Our main axiom states that the transfer leading to X'' is doubly offensive because it increases both income inequality and, more importantly, inequality between groups. Consequently, X'' is less desirable than X' and both induce lower social welfare than X. Notice that, the within group regressive transfer leading to X' is assumed not to alter the order between group, however this order may be defined.[11] The following axiom formalizes our fairness criterion.

Axiom 6. *Inequality aversion (INEQ) - For all $X, X', X'' \in D$, if there exist two types $i, j \in N$ such that $x_{j[r]} \geq x_{i[r]}$ for all $r \in M$ and $\delta > 0$ such that $x_{j[k]} \geq x_{j[l]} \geq x_{j[l]} - \delta \geq x_{i[l]}$, and such that $x''_{j[k]} = x'_{j[k]} = \big(x_{j[k]} + \delta\big)$, $x'_{j[l]} = \big(x_{j[l]} - \delta\big)$, and $x''_{i[l]} = \big(x_{i[l]} - \delta\big)$, with X and X', and X and X'', coinciding everywhere else, then $W(X) \geq W(X') > W(X'')$.*

The six axioms allow us to characterize a family of social welfare functions that first aggregates each group's income distribution through a symmetric and inequality-averse function and then aggregates the values of those functions expressing inequality aversion.

Theorem 1. *For all $X \in D$, the function $W : D \rightarrow \mathbb{R}_+$ satisfies MON, NORM,*

[11] In line with SYM, our model abstains from assuming an exogenous pre-order of groups. Groups may be ordered endogenously, according to their relative income distribution. It is however natural, we believe, to assume that any acceptable order should be consistent with the partial one induced by first order stochastic dominance.

IND, SYM, WSYM and INEQ if and only if

$$W(X) = F\left(f\left(\mathbf{x}_1\right), \ldots, f\left(\mathbf{x}_n\right)\right) \tag{5}$$

where $F : \mathbb{R}^n_+ \to \mathbb{R}_+$ *and* $f : \mathbb{R}^m_+ \to \mathbb{R}_+$ *are both differentiable, strictly monotone, homogeneous of degree one such that* $F(e, \ldots, e) = f(e, \ldots, e) = e$*,* f *is Schur-concave,* F *is strictly Schur-concave and* F *is more Schur-concave than* f*.*

The proof of this theorem can be found in Appendix B.

The literature however is often interested in social welfare functions with an additive structure. We formalize this requirement in the following axiom.

Axiom 7. *Additivity (ADD) - For all* $X \in D$*, there exist* $\tau : \mathbb{R}_+ \to \mathbb{R}_+$*,* $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ *and* $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ *such that* $W(X) = \tau\left(\sum_{j=1}^{n} \psi\left(\sum_{k=1}^{m} \phi\left(x_{jk}\right)\right)\right)$.

We then have the following characterization result.

Proposition 1. *For all* W *as in Eq. (1),* W *satisfies ADD if and only if there exist* $q \leq p \leq 1$ *such that, for all* $X \in D$*,* $W(X) = W_{q,p}(X)$ *where*

$$W_{q,p}(X) = \left[\sum_{j=1}^{n} \frac{1}{n}\left(\frac{1}{m}\sum_{k=1}^{m} x_{jk}^{p}\right)^{q/p}\right]^{1/q}. \tag{6}$$

The proof of this proposition can be found in Appendix B.

B Proofs

B.1 Theorem 1

(ONLY IF)

Let I denote the $n \times m$-dimensional matrix of ones, and $\mathbf{1}_t$ the t-dimensional vector of ones. For all $\mathbf{x}_i \in \mathbb{R}_+^m$, by MON, we can find $a, A \in \mathbb{R}_{++}$ such that

$$W\left(a\mathbf{x}_i, I_{-i}\right) < W\left(\mathbf{1}_m, I_{-i}\right) < W\left(A\mathbf{x}_i, I_{-i}\right).$$

By the Intermediate Value Theorem, there exist a value $f_i\left(\mathbf{x}_i\right)$ such that:

$$W\left(\mathbf{x}_i, I_{-i}\right) = W\left(f_i(\mathbf{x}_i)\mathbf{1}_m, I_{-i}\right).$$

By MON, this value is unique. Next, using SYM twice, we have:

$$W\left(f_i(\mathbf{x})\mathbf{1}_m, I_{-i}\right) = W\left(\mathbf{x}, I_{-i}\right) = W(\mathbf{x}, I_{-j}) = W\left(f_j(\mathbf{x})\mathbf{1}_m, I_{-j}\right) = W\left(f_j(\mathbf{x})\mathbf{1}_m, I_{-i}\right).$$

By MON, we get that $f_i(\mathbf{x}) = f_j(\mathbf{x})$, so that f_i is independent of i. Also, it is easily established that $f(\mathbf{1}_m) = f(1, ..., 1) = 1$ and, by MON, f is strictly increasing.

We now show that $f(.)$ is continuous.

Towards a contradiction, let $\mathbf{x}_t \to \mathbf{x}$ (in $\mathbb{R}_+^m$) and assume that $f(\mathbf{x}_t)$ does not converge to $f(\mathbf{x})$. Let us first show that the sequence $f(\mathbf{x}_t)$ is bounded. As $\mathbf{x}_t$ converges, it is bounded (in the vector sense) by some vectors $\underline{\mathbf{x}}$ and $\overline{\mathbf{x}}$. Then

$$W(f(\mathbf{x}_t)\mathbf{1}_m, I_{-i}) = W(\mathbf{x}_t, I_{-i}) \leq W(\overline{\mathbf{x}}, I_{-i}) = W(f(\overline{\mathbf{x}})\mathbf{1}_m, I_{-i}).$$

By montonicity, $f(\mathbf{x}_t) \leq f(\overline{\mathbf{x}})$. Similarly, we can show that $f(\mathbf{x}_t) \geq f(\underline{\mathbf{x}})$ for all t. Since $f(\mathbf{x}_t)$ does not converge, there is a ε such that, along some subsequence, $|f(\mathbf{x}_t) - f(\mathbf{x})| > \varepsilon$. Without loss of generality, take (if necessary) a further subsequence such that $f(\mathbf{x}_t) > f(\mathbf{x}) + \varepsilon$. Finally, take once again (if necessary) a subsequence such that $f(\mathbf{x}_t)$ converges (which we may do as the sequence is bounded, cfr. Bolzano Weierstrass Theorem).

Then, by continuity of W and MON:

$$\begin{aligned} W(\mathbf{x}, I_{-i}) &= \lim_{t\to\infty} W(\mathbf{x}_t, I_{-i}), \\ &= \lim_{t\to\infty} W(f(\mathbf{x}_t)\mathbf{1}_m, I_{-i}), \\ &\geq W((f(\mathbf{x}) + \varepsilon)\mathbf{1}_m, I_{-i}), \\ &> W(f(\mathbf{x}), I_{-i}) = W(\mathbf{x}, I_{-i}), \end{aligned}$$

the desired contradiction. Since W is assumed to be differentiable, $f(.)$ must be differentiable as well.

We now show that $f()$ is homogeneous of degree one. Observe that $W(\mathbf{x}, I_{-i}) = W(f(\mathbf{x})\mathbf{1}_m, I_{-i})$ and $W(\lambda\mathbf{x}, I_{-i}) = W(f(\lambda\mathbf{x})\mathbf{1}_m, I_{-i})$. By the homogeneity property implied by IND, $W(\lambda\mathbf{x}, I_{-i}) = W(\lambda f(\mathbf{x})\mathbf{1}_m, I_{-i})$. As such, by MON, $\lambda f(\mathbf{x}) = f(\lambda\mathbf{x})$ as desired.

The next step of the proof consists in showing that W is an aggregator of the equally distributed equivalents of each type. That is, there exist an F such that $F\left(f(\mathbf{x}_1), ..., f(\mathbf{x}_n)\right) = W\left(\mathbf{x}_1, ..., \mathbf{x}_n\right))$.

Let $X, X' \in D$ be two distributions, we need to show that if $f(\mathbf{x}_i) = f(\mathbf{x}'_i)$ for all $i \in C$ then $W(X) = W(X')$. So assume that $f(\mathbf{x}_i) = f(\mathbf{x}'_i)$ for all $i \in C$. We know that for all i, $W(\mathbf{x}_i, I_{-i}) = W(f(\mathbf{x}_i)\mathbf{1}_m, I_{-i}) = W(f(\mathbf{x}'_i)\mathbf{1}_m, I_{-i}) = W(\mathbf{x}'_i, I_{-i})$.

Define $\widetilde{X}^j$ the distribution where $\widetilde{\mathbf{x}}_i^j = \mathbf{x}_i$ if $i \leq j$ and $\widetilde{\mathbf{x}}_i^j = \mathbf{x}'_i$ if $i > j$. So $(\mathbf{x}_j, \widetilde{X}^j_{-j}) = \widetilde{X}^j$ and $(\mathbf{x}'_j, \widetilde{X}^j_{-j}) = \widetilde{X}^{j-1}$. By IND, we have that:

$$\begin{aligned} W(\widetilde{X}^i) &= W(\mathbf{x}_i, \widetilde{X}^i_{-i}) = W(f(\mathbf{x}_i)\mathbf{1}_m, \widetilde{X}^i_{-i}) \\ &= W(f(\mathbf{x}'_i)\mathbf{1}_m, \widetilde{X}^i_{-i}) = W(\mathbf{x}'_i, \widetilde{X}^i_{-i}) \\ &= W(\widetilde{X}^{i-1}). \end{aligned}$$

Repeated applications gives $W(X) = W(\widetilde{X}^n) = W(\widetilde{X}^0) = W(X')$.

We now prove that the function F is continuous.

As f is homogeneous of degree one, we have that $f(\lambda, \lambda, \ldots, \lambda) = \lambda f(\mathbf{1}_m)$. Let $\mathbf{z}_t$ in $\mathbb{R}^n_+$ converge to $\mathbf{z}$ as t converges to infinity. Then, as $f(\mathbf{1}_m) = 1$ and f is homogeneous of degree 1,

$$\mathbf{z}_t = (z_{1,t} f(\mathbf{1}_m), ..., z_{n,t} f(\mathbf{1}_m)) = (f(z_{1,t}, ..., z_{1,t}), ..., f(z_{n,t}, ..., z_{n,t}))$$

let Z_t be the distribution where entry in row j is given by $z_{j,t}$ and let Z be the distribution where every entry in row j is given by the jth element of z, say z_j.

By continuity of W,

$$\begin{aligned}\lim_{t\to\infty} F(\mathbf{z}_t) &= \lim_{t\to\infty} F\left(f(z_{1,t},...,z_{1,t}),...,f(z_{n,t},...,z_{n,t})\right)\\ &= \lim_{t\to\infty} W(Z_t),\\ &= W(Z),\\ &= F\left(f(z_1,...,z_1),...,f(z_m,...,z_m)\right),\\ &= F(\mathbf{z}).\end{aligned}$$

the desired outcome.

We can easily show that F is also strictly increasing. Moreover, since f is homogeneous of degree one, we can show that for any n dimensional vectors $\mathbf{z},\mathbf{z}'$, $F(\mathbf{z}) = F(\mathbf{z}')$ if and only if for all $\lambda > 0$, $F(\lambda\mathbf{z}) = F(\lambda\mathbf{z}')$. Now, for $\mathbf{z} \in \mathbb{R}^n_+$, define $G(\mathbf{z})$ to be the unique value such that:

$$F(\mathbf{z}) = F(G(\mathbf{z})\,\mathbf{1}_n).$$

Existence of G follows from the Intermediate Value Theorem and uniqueness from MON. G can be shown to be continuous (similar as for the function f previously). Let us show that G is homogeneous of degree 1. As shown before

$$F(\mathbf{z}) = F\left(\underbrace{G(\mathbf{z})\mathbf{1}_n}_{\mathbf{z}'}\right) \iff F(\lambda\mathbf{z}) = F\left(\lambda\,\underbrace{G(\mathbf{z})\mathbf{1}_n}_{\mathbf{z}'}\right),$$

and, by definition, $F(\lambda\mathbf{z}) = F(G(\lambda\mathbf{z})\mathbf{1}_n)$. Therefore, $\lambda G(\mathbf{z}) = G(\lambda\mathbf{z})$ as desired.

We now show that the ranking of distributions induced by the equally distributed equivalent G represents represents the one of W.

Assume that $W(X) \leq W(X')$,

$$F\left(G\left((f(\mathbf{x}_1), ..., f(\mathbf{x}_n))\mathbf{1}_n\right)\right) = F\left(f(\mathbf{x}_1), ..., f(\mathbf{x}_n)\right) = W(X)$$
$$\leq W(X') = F\left(f(\mathbf{x}'_1), ..., f(\mathbf{x}'_n)\right) = F\left(G\left((f(\mathbf{x}'_1), ..., f(\mathbf{x}'_n))\mathbf{1}_n\right)\right).$$

By monotonicity $G(f(\mathbf{x}_1), ..., f(\mathbf{x}_n)) \leq G(f(\mathbf{x}'_1), ..., f(\mathbf{x}'_m))$.

The proof of the reverse is very similar. We can therefore restrict the function F to coincide with the equally distributed equivalent G.

We only need to prove the last part of the theorem about the Schur-concavity of f and F.

Let δ_k denote the m-dimensional vector of zeroes with k-th entry equal to $\delta > 0$. INEQ, in the $W(X) \geq W(X')$ part, requires that $f\left(\mathbf{x}_j + \delta_k - \delta_l\right) \leq f\left(\mathbf{x}_j\right)$, whenever $(\mathbf{x}_j + \delta_k - \delta_l)$ corresponds to $\mathbf{x}_j$ after a mean-preserving regressive transfer. This is always the case only if f is Schur-concave.

Similarly, in the $W(X) > W(X'')$ part, INEQ implies that F is strictly Schur-concave. To see this, let $\mathbf{a}, \mathbf{b} \in \mathbb{R}^m_+$ be such that $\mathbf{a} = (a, ..., a)$, $\mathbf{b} = (b, ..., b)$, $a > b$. Then, let $\delta > 0$, by INEQ

$$F\left(f(\mathbf{a}), f(\mathbf{b}), \mathbf{1}_{n-2}\right) < F\left(f(\mathbf{a} + \mathbf{1}_m\delta), f(\mathbf{b} - \mathbf{1}_m\delta), \mathbf{1}_{n-2}\right)$$

that is $F(a, b, 1, .., 1) < F(a + \delta, b - \delta, 1, .., 1)$, which is the case if and only if F is strictly Schur-concave.

Let $E \in \mathbb{R}^{n\times n}_{++}$ be such that $e_{jk} = e$ for all $(j, k) \in \{1, .., n\}^2$, $X \in \mathbb{R}^{n\times n}_{++}$ be such that $x_{jk} = x_{jl} = e_j$ for all $(j,k),(j,l) \in \{1,..,n\}^2$, and $Y \in \mathbb{R}^{n\times n}_{++}$ be such that $\mathbf{y}_j = (e_1,..,e_n)$ for all $j \in \{1,..,n\}$. [12]

Notice that X can be obtained from E through a series of regressive between type transfers while Y can be obtained from E through a series of regressive within type transfers. By NORM, we have $W(Y) = F[f(e_1,..,e_n),..,f(e_1,..,e_n)] = f(e_1,..,e_n)$ and $W(X) = F[f(e_1,..,e_1),..,f(e_n,..,e_n)] = F(e_1,..,e_n)$.

Since INEQ implies $W(Y) \geq W(X)$, we have $f(e_1,..,e_n) \geq F(e_1,..,e_n)$ so that, F is more Schur-concave than f.

(IF)

It is easy to see that, by definition, Eq. 5 satisfies MON, NORM, SYM, WSYM and IND. We only need to show that it satisfies INEQ.

Let the distribution X be such that $x_{jk} = x_{j[k]}$ for all $(j,k) \in N \times M$,[13] and let it satisfy the conditions in INEQ. In particular, let $\mathbf{x}_j \geq \mathbf{x}_h$, for $j,h \in C$, and let $\delta > 0$ be such that $x_{jk} \geq x_{jt} \geq x_{jt} - \delta \geq x_{ht}$ for $k,t \in R$.

Denote δ_i the m-dimensional vector of zeroes with value δ at the i-th entry. The first step of the proof consists in showing that

$$0 \geq f(\mathbf{x}_j + \delta_k - \delta_t) - f(\mathbf{x}_j) \geq f(\mathbf{x}_j + \delta_k) - f(\mathbf{x}_j) + f(\mathbf{x}_h - \delta_t) - f(\mathbf{x}_h) \quad (7)$$

The first inequality is obvious. The second one is direct consequence of the following claim, after setting $y = \mathbf{x}_h$ and $c = \mathbf{x}_j + \delta_k - \mathbf{x}_h \geq 0$.

[12]An example for the case $n = 2$ is $E = \begin{bmatrix} 15 & 15 \\ 15 & 15 \end{bmatrix}; X = \begin{bmatrix} 10 & 10 \\ 20 & 20 \end{bmatrix}; Y = \begin{bmatrix} 10 & 20 \\ 10 & 20 \end{bmatrix}$

[13]In other words, we assume types to be increasingly ordered. This assumption is without loss of generality and allows us to simplify notation.

Claim 1 Let $f : \mathbb{R}_+^m \to \mathbb{R}_+$ be a monotone, differentiable, equally distributed equivalent, homogeneous of degree one and Schur-concave function. For all $y, c \in \mathbb{R}_+^m$ and all $\delta_t \in \mathbb{R}_+^m$, where δ_t is a vector of zeroes with value $\delta > 0$ at the entry t,

$$f(y + c - \delta_t) - f(y + c) \geq f(y - \delta_t) - f(y) \tag{8}$$

Proof of Claim 1

Observe that if $c = 0$, the result follows trivially. If f is linear, so that it has constant partial derivatives, we can use the Euler's Theorem of homogeneous functions to prove that (8) holds with equality.

We only need to prove this for f non-linear. Let us assume that f is non-linear. Towards a contradiction assume that, for some δ_t, we have

$$f(y + c) - f(y + c - \delta_t) > f(y) - f(y - \delta_t)$$

so that

$$\frac{f(y + c) - f(y + c - \delta_t)}{\delta} > \frac{f(y) - f(y - \delta_t)}{\delta} \tag{9}$$

which corresponds to $\partial_t f(y+c) \geq \partial_t f(y)$, where $\partial_t f(y)$ denotes the partial derivative of $f(y)$ with respect to the t-th entry (cfr. Lagrange Theorem). By assumption, f is symmetric, so the previous inequality holds for all $t = 1, ..., m$. Consequently, f must be a convex function.

We now have that f is symmetric and convex, hence Schur-convex.

Suppose f is both Schur-concave and Schur-convex. Then for any $x, y \in \mathbb{R}_+^n$

such that x is majorized by y, we have $f(x) \leq f(y) \leq f(x)$, which implies $f(x) = f(y)$. In particular, f has to be constant on each trace hyperplane $T_c := \left\{x \in \mathbb{R}_+^n : \sum_{j=1}^n x_j = c\right\}$, $c \in \mathbb{R}_+$ because $\frac{c}{n}(1, 1, ..., 1) \prec x$ for all $x \in T_c$.

This allows us to define a mapping $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ as $\alpha(c) := f\left(\frac{c}{n}(1, 1, ..., 1)\right)$ to see that, for all $x \in \mathbb{R}_+^n$,

$$f(x) = f\left(\frac{\sum_{j=1}^n x_j}{n}(1, 1, ..., 1)\right) = \alpha\left(\sum_{j=1}^n x_j\right).$$

Observe now that, for f to be equally distributed equivalent, we should have $\alpha\left(\sum_{j=1}^n x_j\right) = \frac{1}{n}\sum_{j=1}^n x_j$. Consequently,

$$f(x) = \frac{1}{n}\sum_{j=1}^n x_j$$

so that f is a linear function. The desired contradiction.

This concludes the proof of our claim.

Without loss of generality, let $j = 1$ and $h = 2$. The second step of the proof consists in showing the following inequalities:

$$\begin{aligned} &F\left[f(\mathbf{x}_1 + \delta_k), f(\mathbf{x}_2 - \delta_t), f(\mathbf{x}_3), ..., f(\mathbf{x}_n)\right] \leq \\ &\qquad F\left[f(\mathbf{x}_1 + \delta_k - \delta_t), f(\mathbf{x}_2), f(\mathbf{x}_3), ..., f(\mathbf{x}_n)\right] \leq \\ &\qquad\qquad F\left[f(\mathbf{x}_1), f(\mathbf{x}_2), f(\mathbf{x}_3), ..., f(\mathbf{x}_n)\right] \quad (10) \end{aligned}$$

Let $\Delta_1^B = f(\mathbf{x}_1 + \delta_k) - f(\mathbf{x}_1)$, $\Delta_2^B = f(\mathbf{x}_2) - f(\mathbf{x}_2 - \delta_t)$ and $\Delta_1^W = f(\mathbf{x}_1) -$

$f(\mathbf{x}_1 + \delta_k - \delta_t)$. With this notation, Eq. 7 becomes $0 \geq -\Delta_1^W \geq \Delta_1^B - \Delta_2^B$, that is $\Delta_2^B \geq \Delta_1^B + \Delta_1^W \geq 0$. Let us also recall that $f(\mathbf{x}_1 + \delta_k - \delta_t) \geq f(\mathbf{x}_2)$. The following (in)equalities are direct consequence of all this, plus Schur-concavity and monotonicity of F.

$$
\begin{gathered}
F\left[f\left(\mathbf{x}_1+\mathbf{1}_k \delta\right), f\left(\mathbf{x}_2-\mathbf{1}_q \delta\right), f\left(\mathbf{x}_3\right), \ldots, f\left(\mathbf{x}_n\right)\right]= \\
F\left[f\left(\mathbf{x}_1\right)+\Delta_1^B, f\left(\mathbf{x}_2\right)-\Delta_2^B, f\left(\mathbf{x}_3\right), \ldots, f\left(\mathbf{x}_3\right), \ldots, f\left(\mathbf{x}_n\right)\right] \leq \\
F\left[f\left(\mathbf{x}_1\right)+\Delta_1^B, f\left(\mathbf{x}_2\right)-\Delta_1^B-\Delta_1^W, f\left(\mathbf{x}_3\right), \ldots, f\left(\mathbf{x}_3\right), \ldots, f\left(\mathbf{x}_n\right)\right]= \\
F\left[f\left(\mathbf{x}_1\right)+\Delta_1^B+\Delta_1^W-\Delta_1^W, f\left(\mathbf{x}_2\right)-\Delta_1^B-\Delta_1^W, f\left(\mathbf{x}_3\right), \ldots, f\left(\mathbf{x}_3\right), \ldots, f\left(\mathbf{x}_n\right)\right] \leq \\
F\left[f\left(\mathbf{x}_1\right)-\Delta_1^W, f\left(\mathbf{x}_2\right), f\left(\mathbf{x}_3\right), \ldots, f\left(\mathbf{x}_3\right), \ldots, f\left(\mathbf{x}_n\right)\right]= \\
F\left[f\left(\mathbf{x}_1+\mathbf{1}_k \delta-\mathbf{1}_t \delta\right), f\left(\mathbf{x}_2\right), f\left(\mathbf{x}_3\right), \ldots, f\left(\mathbf{x}_3\right), \ldots, f\left(\mathbf{x}_n\right)\right] \leq \\
F\left[f\left(\mathbf{x}_1\right), f\left(\mathbf{x}_2\right), f\left(\mathbf{x}_3\right), \ldots, f\left(\mathbf{x}_3\right), \ldots, f\left(\mathbf{x}_n\right)\right] \quad (11)
\end{gathered}
$$

The desired outcome.

B.2 Proof of Proposition 1

It is easily shown that $W_{q,p}$ satisfies ADD. Observe that if W satisfies ADD, then, for all $j \in C$, we can write $f(\mathbf{x}_j) = g\left(\sum_{k=1}^m \phi(x_{jk})\right)$ for some increasing, differentiable and concave function ϕ (Marshall *et al.*, 2011, ch. 3), and a function g that preserves NORM. Since f is equally distributed equivalent, and homogeneous of degree one, we have that, for all $j \in C$, the ratio between $f(\mathbf{x}_j)$ and the average income in $\mathbf{x}_j$ is invariant to equal proportional changes in all incomes. As also shown in Atkinson (1970),[14] this is equivalent to having $f(\mathbf{x}_j) = \left(\frac{1}{m}\sum_{k=1}^{m} x_{jk}^p\right)^{1/p}$ for $p \leq 1$. Applying the same reasoning to F we get the desired result.

B.3 Dominance conditions

For all $X \in D$, call $X^{\downarrow}$ the row-permutation of X such that $\mu(\mathbf{x}_1^{\downarrow}) \leq \mu(\mathbf{x}_2^{\downarrow}) \leq \ldots \leq \mu(\mathbf{x}_n^{\downarrow})$; in words, $X^{\downarrow}$ orders the rows in X according to their average. For all $X \in D$, let us also define a standardized distribution $\hat{X} \in D$ such that, for all $(j,k) \in N \times M$, $\hat{x}_{jk} = x_{jk}^{\downarrow}/\mu(\mathbf{x}_j^{\downarrow})$; that is, $\hat{X}$ normalizes the rows of $X^{\downarrow}$ with respect to their average. With this notation, we can introduce a test to perform robust partial comparisons of income distributions.

Proposition 2. *For all* $X, Y \in D$, *if* (i) $L(\hat{X}) \geq L(\hat{Y})$ *and* (ii) $gL(X_\mu) \geq gL(Y_\mu)$, *then* $W(X) \geq W(Y)$ *for all* W *as in Eq. (1).*

Proof. Observe that, by symmetry of F and degree's one homogeneity of f, for all $X \in D$, W as in Eq. (1)

$$W(X) = W(X^{\downarrow}) = F\left[\mu\left(\mathbf{x}_1^{\downarrow}\right) f\left(\hat{\mathbf{x}}_1\right), ..., \mu\left(\mathbf{x}_n^{\downarrow}\right) f\left(\hat{\mathbf{x}}_n\right)\right]$$

Hence, $W(X) \geq W(Y)$ for all W is equivalent to

$$F\left[\mu\left(\mathbf{x}_1^{\downarrow}\right) f\left(\hat{\mathbf{x}}_1\right), ..., \mu\left(\mathbf{x}_n^{\downarrow}\right) f\left(\hat{\mathbf{x}}_n\right)\right] \geq F\left[\mu\left(\mathbf{y}_1^{\downarrow}\right) f\left(\hat{\mathbf{y}}_1\right), \ldots, \mu\left(\mathbf{y}_n^{\downarrow}\right) f\left(\hat{\mathbf{y}}_n\right)\right] \quad (12)$$

for all F and f as in Theorem 1.

[14] See also Theorem 4.2 in Lambert (1992)

Let $X, Y \in D$ satisfy conditions (i) and (ii) in the Proposition. We need to show that eq. 12 holds true. Let us write (ii) as

$$\frac{1}{n}\sum_{j=1}^{k}\mu(\mathbf{x}_j^{\downarrow}) \geq \frac{1}{n}\sum_{j=1}^{k}\mu(\mathbf{y}_j^{\downarrow}) \qquad \forall k = 1, \ldots, n$$

and observe that, by S-concavity of f, (i) is equivalent to

$$f\left(\hat{\mathbf{x}}_j\right) \geq f\left(\hat{\mathbf{y}}_j\right) \qquad \forall j \in C$$

Following Shorrocks (1983), eq. 12 is satisfied if and only if

$$\frac{1}{n}\left[\sum_{j=1}^{k}\mu(\mathbf{x}_j^{\downarrow})f(\hat{\mathbf{x}}_j) - \sum_{j=1}^{k}\mu(\mathbf{y}_j^{\downarrow})f(\hat{\mathbf{y}}_j)\right] \geq 0 \qquad \forall k = 1, \ldots, n$$

Let $\epsilon_j = f\left(\hat{\mathbf{x}}_j\right) - f\left(\hat{\mathbf{y}}_j\right)$ for all $j \in C$, and observe that (i) implies $e_j \geq 0$ for all j. We can rewrite the previous condition as

$$\frac{1}{n}\sum_{j=1}^{k}\left[\mu(\mathbf{x}_j^{\downarrow})\left(f\left(\hat{\mathbf{y}}_j\right) + \epsilon_j\right) - \mu(\mathbf{y}_j^{\downarrow})f\left(\hat{\mathbf{y}}_j\right)\right] \geq 0 \qquad \forall k = 1, \ldots, n$$

$$\frac{1}{n}\sum_{j=1}^{k}f(\hat{\mathbf{y}}_j)\left[\mu(\mathbf{x}_j^{\downarrow}) - \mu(\mathbf{y}_j^{\downarrow})\right] + \frac{1}{n}\sum_{j=1}^{k}\epsilon_j\mu(\mathbf{x}_j^{\downarrow}) \geq 0 \qquad \forall k = 1, \ldots, n$$

It is now sufficient to notice that (ii) ensures the first sum to be positive, and, by (i), also the second term of the inequality is positive. The desired result. $\square$

C Descriptive Statistics

Table 1: Summary Statistics of Income by Year in US

Year	Mean	SD	Min	Median	Max	N
1970	29.378	19.102	0.439	25.248	200.279	4112
1972	31.656	20.706	0.213	27.282	227.216	4524
1974	32.084	20.675	0.005	27.680	306.702	4842
1976	32.388	19.765	0.008	28.630	217.416	5068
1978	32.719	21.357	0.075	28.979	460.359	5253
1980	31.067	36.892	0.665	26.156	1593.111	5470
1982	29.415	20.401	0.108	25.666	325.902	5662
1984	31.963	28.428	0.039	26.735	703.821	5794
1986	32.405	25.959	0.061	27.091	403.277	5803
1988	36.783	48.495	0.363	28.745	1781.267	5950
1990	35.279	32.714	0.019	28.690	696.315	6092
1992	36.233	37.968	0.033	28.825	807.481	6189
1994	35.645	37.631	0.009	28.204	1145.452	6810
1998	40.082	60.178	0.012	31.840	3559.347	6707
2000	42.228	42.207	0.027	33.151	874.269	6599
2002	43.071	64.383	0.029	33.311	2265.240	6787
2004	42.348	60.186	0.019	33.287	3371.559	8217
2006	41.266	52.172	0.021	31.054	1783.123	8289
2008	41.112	47.941	0.009	32.411	1872.444	8478
2010	39.920	42.555	0.016	31.572	922.733	8346
2012	40.669	45.961	0.002	32.069	1747.119	8164
2014	40.777	37.495	0.014	32.404	645.899	7769

Description: The table reports summary statistics for equivalised disposable household income in dollars PPP 2017. Values are divided by 1,000. *Source:* Own elaboration based on PSID data.

Table 2: Population shares by ethnic group in the US

Year	White	Black	AmInd	Asian	Hispanic	Other
1970	0.713	0.247	0.013	0.004	0.011	0.011
1972	0.705	0.254	0.015	0.004	0.011	0.011
1974	0.694	0.265	0.016	0.004	0.010	0.011
1976	0.690	0.270	0.015	0.004	0.010	0.011
1978	0.683	0.279	0.016	0.003	0.010	0.010
1980	0.677	0.285	0.016	0.003	0.009	0.010
1982	0.672	0.291	0.016	0.003	0.009	0.009
1984	0.669	0.295	0.017	0.003	0.008	0.008
1986	0.669	0.296	0.017	0.004	0.007	0.008
1988	0.671	0.294	0.017	0.003	0.006	0.008
1990	0.674	0.292	0.017	0.003	0.006	0.008
1992	0.678	0.288	0.017	0.003	0.005	0.008
1994	0.672	0.291	0.018	0.004	0.006	0.009
1998	0.710	0.209	0.018	0.019	0.006	0.038
2000	0.711	0.209	0.018	0.017	0.005	0.040
2002	0.704	0.214	0.017	0.017	0.004	0.044
2004	0.623	0.296	0.019	0.016	0.002	0.043
2006	0.616	0.304	0.018	0.014	0.002	0.045
2008	0.607	0.310	0.018	0.014	0.002	0.048
2010	0.605	0.314	0.017	0.013	0.003	0.049
2012	0.596	0.322	0.016	0.012	0.003	0.051
2014	0.589	0.330	0.016	0.012	0.003	0.051

Description: The table reports the population shares of each ethnic group for each year.

Source: Own elaboration based on PSID data.

Table 3: Summary Statistics of Income by Year and Country in Europe

Year	Country	Mean	SD	Min	Median	Max	N
2005	Germany	35.234	24.109	0.136	30.248	648.993	8257
2005	Spain	21.756	13.968	0.002	19.252	314.890	20165
2005	France	32.378	18.460	0.677	28.728	276.452	12254
2005	Italy	30.706	22.018	0.043	26.663	606.066	34431
2011	Germany	30.352	19.449	0.122	27.372	585.837	9825
2011	Spain	23.642	15.498	0.025	20.785	284.510	15624
2011	France	30.138	27.021	0.077	25.903	944.618	10436
2011	Italy	26.666	23.035	0.005	23.568	1363.660	22555
2019	Germany	33.053	23.473	0.048	29.310	465.249	7354
2019	Spain	22.475	14.457	0.005	19.858	219.746	19315
2019	France	30.283	62.307	0.270	26.424	6040.091	10114
2019	Italy	26.098	15.798	0.011	23.651	211.387	19410

Description: The table reports summary statistics for equivalised annual disposable household income in USD 2010. Values are divided by 1,000.

Source: Own elaboration based on EU-SILC data.

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