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Regional Differences in Italian School Efficiency: A Conditional DEA Approach

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Abstract

This research investigates the intricate interplay among school performance, educational achievements, individual backgrounds, and regional differences within Italian secondary education. Drawing on data from the 2021-2022 Invalsi database, the study focuses on final-year high school students, examining the relationship between Standardized Invalsi test scores and Official grades in Italian and Mathematics. The primary goal is to assess school efficiency using a Conditional Data Envelopment Analysis (DEA) approach. Additionally, the paper explores regional differences in school performance through the application of the Theil Index. Critical determinants of school performance, such as school size and socioeconomic status, are identified. The findings highlight significant regional disparities, revealing that schools in the north of Italy outperform those in the south, particularly in Invalsi scores. Moreover, certain schools tend to excel in official grades relative to their Invalsi scores, indicating the need for policies to address these regional educational inequalities and improve school performance across Italy.

Keywords: School Efficiency, Conditional DEA, Regional Differences, Educational Achievements, Invalsi

JEL Codes: I21, I24, I26

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1 Introduction

The education sector encompasses all activities related to teaching, training, and learning across various disciplines, involving students, teachers, families, and institutions. This process involves multiple actors, and for this reason, the analysis of education efficiency is a widely discussed topic in political and civic debates as it involves a crucial aspect of society (Witte & López-Torres (2017)).
This paper aims to (i) assess the efficiency of the Italian secondary school system, (ii) identify what are the most influencing factors of school performance factors influencing school efficiency, and finally, (iii) examine divergences at the regional level in Italy.

The purpose of evaluating school efficiency is to analyze individual performance, establish new goals, decide on future resource allocations, and enhance the overall performance of school operations (Soteriou *et al.* (1998)). Measuring efficiency in education generally implies that, on the one hand, students' characteristics and school organizational features are considered as inputs. On the other hand, students' educational achievements represent the outputs of the production function. Emrouznejad & Thanassoulis (1996) defines three variables that primarily determine the level of a school's efficiency. The first involves specific school aspects (number and quality of teachers, class size); the second variable relates to student characteristics and their abilities or inclination to study. Finally, the third variable encompasses the external environment and the student's family. It is important to underline how a school's performance is strongly influenced by factors that are not directly controllable by the school system, such as the personal socioeconomic background of each student, as highlighted in the literature by Afonso & Aubyn (2006)) and Agasisti & Zoido (2019)). Naturally, these uncontrollable heterogeneous factors must be adequately weighted in the analyses so that objective policies can be suggested and defined Mergoni & De Witte (2022)).

Efficiency assessment can be measured using various approaches: parametric, non-parametric, deterministic, and stochastic. Data Envelopment Analysis (DEA) is a deterministic non-parametric method commonly used in assessing school efficiency. DEA ignores noise in both the data and the model, which is characterized by a highly flexible non-parametric structure and limited assumptions for the distribution of inefficiencies. However, the traditional DEA model suffers from endogeneity and does not fully utilize all the information from the production process of each Decision Making Unit (DMU), often resulting in distortions (Cook *et al.* (2010)). Moreover, Simar & Wilson (2011) identify other three limits of traditional DEA: (i) the efficiency coefficient is artificially anchored at 1, (ii) the input-to-output production function correlation with efficiency estimation is highly complex, (iii) efficiency estimates are often correlated with each other. For these reasons, different versions of DEA have been developed, such as conditional DEA by Daraio & Simar (2007)).

The stream of literature regarding education efficiency (Charnes *et al.* (1978); Bessent & Bessent (1980); Charnes *et al.* (1981); Bessent *et al.* (1982),;), is based on the concept that the institution providing education, typically a school, is deemed efficient if the producers optimally utilize the available resources. Schools, in the context of efficiency evaluation, can be treated as service businesses since they generate outputs using inputs (Cameron (1978)).

Indeed, they are excellent examples for assessing efficiency because they are non-profit organizations (Witte & López-Torres (2017)). Therefore, to estimate the efficiency of each school, the economics of education must assess a production function and the technology through which students acquire knowledge (Worthington (2001); Johnes (2015)).
In a school context, the operating framework diverges significantly from corporate contexts, complicating the identification of composite outputs. For instance, a student's proficiency in Mathematics often correlates with their performance in Italian, illustrating the interconnected nature of academic achievement. This interconnection poses a challenge in evaluating school efficiency, as it complicates the formulation of a production function and the a priori determination of parameters (De Witte & Kortelainen (2013)). The wide application of DEA in academic research about education is largely due to its flexibility and the ease with which its findings can be interpreted.

A school is efficient if it produces the maximum output for a given level of input, achieving observed results with the minimum use of resources. Conversely, a school is deemed inefficient if the output level falls below the predefined result for that input quantity (Aubyn *et al.* (2009)). Furthermore, as emphasized by Agasisti & Soncin (2021)), the school production function consists of transforming many inputs into multiple outputs.
Efficiency cannot be separated from effectiveness. The former means doing things well, while the latter means reaching a purpose. This applies to education as well because an acceptable level of desired outcomes will always exist, being on the effectiveness frontier that can be achieved (Witte & López-Torres (2017)). Hence, the existence of efficiency implies effectiveness, and the two concepts cannot be separated, as their combination proves to be an excellent tool for evaluating public policies and identifying inefficiencies in public sectors, such as schools (Mergoni & De Witte (2022)).

The efficiency of Italian schools has been analyzed by several authors who utilized both Invalsi and Pisa data [1]. These studies have widely highlighted significant differences in academic performance among schools, particularly in Mathematics, Italian, and English. Moreover, many of these disparities seem to be closely linked to students' gender and their geographical region of origin.
Di Giacomo & Pennisi (2015) demonstrates how territorial characteristics influence not only students' academic achievements but also schools' performances. Specifically, it has emerged that schools located in northern Italy tend to exhibit better performance compared to those in the South. These conclusions are supported by other significant studies confirming the relevance of geographical variability on school performances (Agasisti & Vittadini (2012); Agasisti & Cordero-Ferrera (2013); Camanho *et al.* (2021); Daniele (2021); Agasisti & Porcelli (2023)) .

By employing different methodologies, this paper shows a comparison between the distribution of Official Grades and the distribution of Invalsi Scores in defining the efficient performance of Italian secondary schools. The data are extracted from the Invalsi database for the academic year 2021-2022 and specifically pertain to students attending their last year

[1] Cipollone *et al.* (2010); Agasisti *et al.* (2014);Masci *et al.* (2018)

of high school.
The first methodology applied is the Data Envelopment Analysis (DEA) to evaluate school efficiency based on inputs (hours spent at school) and outputs (including test scores and grades). It is employed in both traditional and conditional versions, where the latter enables the consideration of environmental variables. Additionally, the analysis delves into regional disparities by leveraging the Theil Index, providing insights into disparities in schools' performances within and between regions. The findings stress the influence of socioeconomic status, school type, and size on school efficiency, observing that Northern Italian schools exhibit higher efficiency than their Southern counterparts. Regional disparities notably impact test scores, while gender inequality manifests in Official Grades.

The article is structured as follows: The next section presents the methodology applied. Section 3 describes the data used in the empirical analysis and how it was conducted. The results, however, are explained in Section 4, followed by the concluding part and a proposal of some policy implications.

2 Methodology

2.1 DEA and Conditional-DEA for Efficiency

Data Envelopment Analysis (DEA) is a deterministic and empirical methodology for assessing performance. introduced it citefarrell1957measurement) but became popular with the paper by Charnes *et al.* (1978) and is therefore also known as the CCR Approach.
DEA is a mathematical optimization technique used to assess the performance of various entities such as nations, schools, service enterprises, and hospitals, which are called Decision Making Units (DMU). It works as a non-parametric linear programming method, using linear combinations of weighted inputs and outputs to establish an efficient frontier. In this process, each DMU aims to maximize its performance. A DMU is considered efficient if it is relatively good at producing output given its input level. The adjective relative means that each DMU will be compared with any other homogeneous unit. On the other hand, DMUs deemed inefficient can improve their performance either by increasing current output levels or decreasing input levels.

As Stolp (1990)) points out, the choice of inputs and outputs strongly influences the assessment of efficiency for individual DMUs. The value of the efficiency coefficient changes depending on the number of variables considered, and consequently, if the sum of the number of inputs and outputs considered is equal to the total number of DMUs analyzed, they will all be considered efficient. To overcome this problem, Becker *et al.* (1964) defines that the minimum optimal number of DMUs to be considered for the DEA Approach is at least three times the sum of inputs and outputs considered in the analysis.
The strong sensitivity to change of the DEA approach, due to its non-parametric form, turns out to be an econometric problem rather than a production problem. Therefore, this also occurs because of its sensitivity to outliers.

There are three types of DEA: (i) *Input-Oriented model*, in which each unit maintains the same output level and minimizes the input level. (ii) *Output-Oriented model*, each DMU maximizes the output level while maintaining the input level. (iii) *Radial model*, in which an attempt is made to mark the minimum distance determined by the increase in output and the reduction in input in a proportion in which the combination turns out to be the best. In this model, slacks represent the potential improvements in input and output variables for the inefficient units in the dataset compared to the other efficient units in the sample. The decision on what type of specification to use is determined by the variables that can be controlled. If one can control inputs, one will proceed with the input-oriented model. Furthermore, the DEA methodology can be customized to various output scales, including constant returns to scale (CRTS) and variable returns to scale (VRTS), such as increasing returns to scale (IRTS) or decreasing returns to scale (DRTS).

However, in the paper, the implemented model is output-oriented with variable returns to scale because it is not known how students perform on subsequent tests. Applying this type of orientation is natural, as it is impossible to control the scores on the first tests if one does not select students based on academic performance.

The DEA approach is popular for evaluating efficiency in school systems primarily due to its flexibility, making it easily adaptable to this field. Its main characteristic is that it does not impose a specific functional form on the production function but determines certain assumptions about the properties of the data to define an efficient frontier (Baker & Matthews (2001); Todd & Wolpin (2003)). Furthermore, as Worthington (2001) suggests, a system with multiple inputs and multi-outputs can be handled.
In the educational sector, applying the DEA approach implies that each school is considered a decision-making unit. In the academic production function, outputs are considered the scores, grades, or votes obtained by students.
As Farrell (1957)) pointed out, DEA is an approach that maximizes outputs given a certain level of inputs. It does not include an allocative efficiency function, whereby the capacity of input utilization is maximized. Therefore, it may be the case that a school is called efficient only because the right combination of inputs and outputs has been calculated, but it may be inefficient. Moreover, highlighting how some inputs are strongly endogenous variables, such as students' socioeconomic background, is important. This, of course, distorts the analysis of efficiency.

Mathematically, the DEA Approach can be written as follows.
In the model[2], $N = \{1, .., n\}$ is the set of DMUs considered in the sample. Every $DMU_j (\forall j \in N)$ uses a vector $X = \{x_1, ..., x_m\} \in \mathbb{R}^m_+$ of m inputs, and produces a vector $Y = \{y_1, ..., y_s\} \in \mathbb{R}^s_+$ of s outputs. Thus, each DMU_j has individual vectors of inputs and outputs (X_j, Y_j) as $x_{ij} (i = 1, .., m)$ and $y_{kj} (k = 1, .., s)$.

The feasible input-output combination defines the set Ψ:

$$\Psi = \{(x, y) \in \mathbb{R}^m_+ \times \mathbb{R}^s_+ | \text{x can produce y}\}. \tag{1}$$

[2]Adapted from Mergoni (n.d.)

The efficiency of each DMU_j is computed by solving the following linear program maximization.

$$\hat{\theta}_{\text{DEA}}(x_0, y_0) = \max \theta \quad \text{s.t.} \tag{2}$$

$$\theta x_{0,i} \geq \sum_{j=1}^{n} \lambda_j X_{j,i} \quad \forall i = 1, \ldots, m$$

$$y_{0,r} \leq \sum_{j=1}^{n} \lambda_j Y_{j,r} \quad \forall r = 1, \ldots, s$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0$$

Where θ is the output-oriented constant return to scale DEA efficiency. $X_{j,i}$ and $Y_{j,r}$ are the i_{th} input and output relative to unit j, respectively. $x_{0,i}$ and $y_{0,r}$ are the input and output of the DMU on which the efficiency score is computed. λ_j is the endogenous weights associated with the inputs and outputs. $\sum_{j=1}^{n} \lambda_j = 1$ means that the return to scale are variable. These weights are determined solely to maximize the efficiency scores of each unit. A DEA approach's deficiency is to attribute deviations from the efficient frontier to inefficiency. This implies that there is no causal noise.

$\hat{\Theta}$ represents the technical efficiency value of the evaluated DMU. When $\hat{\Theta} = 1$, the school is considered efficient (Cooper *et al.* (2007)). An inefficient DMU is indicated by $\hat{\Theta} \geq 1$. The value $(\hat{\Theta} - 1)$ illustrates the additional input needed to achieve the same output as an efficient DMU.

Cazals *et al.* (2002), subsequently Daraio & Simar (2005) and Daraio & Simar (2007) realized the distorted estimates produced by the implementation of a traditional DEA approach, in which variables that are not strictly exogenous, such as family background, school size, or location geographical, are not considered. Environmental factors are a potential source of inefficiency. In the model, $Z \in \mathbb{R}^k_+$ is a matrix of environmental variables, which must be considered in performance measurement.

In the conditional version of the DEA approach, these variables are considered exogenous, and therefore, the separability condition required by traditional DEA is no longer necessary. This separability assumption dictates that environmental variables do not influence the production function involving inputs and outputs. The disadvantages resulting from the application of conditional DEA are mainly twofold: it is a descriptive approach and leaves no room for causal interpretations (Haelermans & De Witte (2012)), and it also has a high computational cost (De Witte & Kortelainen (2013)).

Starting from this probabilistic assumption, Cazals *et al.* (2002), followed by Daraio & Simar (2007)and De Witte & Kortelainen (2013), defined that the consideration of environmental variables $(Z \in \mathbb{R}^k_+)$ in the analysis can be carried out by conditioning the entire

production process on a value of $Z = z$. Moreover, they assume that $(\Omega, \mathcal{A}, \mathbb{P})$ is the probability space on which the variables are defined.
The new set of all feasible input-output combinations becomes:

$$\Psi^z = \{(x, y)|\text{x can produce y when} Z = z\}. \tag{3}$$

The conditional function in the Traditional DEA is:

$$H_{XY}(x, y) = Pr(X \geq x, Y \leq y) \tag{4}$$

Then, the conditional function in the Conditional DEA becomes:

$$H_{XY|Z}(x, y|z) = Pr(X \geq x, Y \leq y|Z = z) \tag{5}$$

Where $H_{XY|Z}(x, y|z)$ is the probability that a unit at level (x, y) can be dominated by other units under the same environmental conditions z. This can be decomposed into:

$$\begin{aligned} H_{XY|Z}(x, y|z) = Pr(Y \leq y|X \geq x, Z = z)Pr(X \geq x, Z = z) \\ = S_{Y|X,Z}(Y \leq y|X \geq x, Z = z)F_x(X \geq x; Z = z) \\ = S_Y(y|x, z)F_x(x|z) \end{aligned} \tag{6}$$

As a result, the output-oriented efficiency coefficient can be obtained:

$$\hat{\theta}(x, y|z) = \sup\left\{\theta > 0 \middle| S_{Y|XZ}(\theta y|X \geq x; Z = z) > 0\right\} \tag{7}$$

Nevertheless, estimating $S_Y(y|x, z)$ becomes more complex in the unconditional case, as it requires implementing smoothing techniques for the variables z.

$$\hat{S}_{Y,n}(y|x, z) = \frac{\sum_{i=1}^{n} I(x_i \leq x, y_i \geq y)K_{\hat{h}}(z, z_i)}{\sum_{i=1}^{n} I(x_i \leq x)K_{\hat{h}}(z, z_i)} \tag{8}$$

This methodology is thus based on estimating the non-parametric Kernel function, which is useful in selecting a reference bandwidth parameter. With this purpose, De Witte & Kortelainen (2013) devised a model where both continuous and discrete variables can be considered, thereby extending the work of Racine & Li (2004) and Hsiao *et al.* (2007). This model is based on the idea of multiplying three different multivariate Kernel functions, one for each type of variable, and obtaining a generalized product of the Kernel functions $K_{\hat{h}}$, to be substituted in equation 14, thereby obtaining a new $\hat{S}_{Y,n}(y|x, z)$ to be substituted in equation 13, thus obtaining a new output-oriented efficiency coefficient $\hat{\theta}(x, y|z)$.

In this context, the *Efficient Production Frontier* is defined as:

$$\hat{\Psi}_Z = \max\{(x, y) \in \mathbb{R}^{m+s} \,|\, y \leq \sum_n \gamma Y, \, x \geq \sum_n \gamma X \text{ per } (\gamma_1, \ldots, \gamma_n) > 0 \text{ e } Z = z\} \tag{9}$$

and the *Conditional Efficiency Coefficient* as:

$$\hat{\Theta}_m(x, y|z) = \int_0^\infty [1 - F(X|Y, Z)(u|y, z)] \, du \tag{10}$$

2.2 Theil Index

The Theil index is derived from Shannon's information theory Shannon (1949), which quantifies inequality by assessing the information entropy in a population distribution (Khinchin (2013); Kullback (1997)).
The notion of entropy refers to the expected information of a given situation. This implies that if n random events (e) are considered, the probability with which they can occur is equal to w. The information inferred from the event is equal to ϕ and $\phi(w)$ is its probability function, which is differentiable and takes the following form:

$$\phi(w) = -log(w) = log(\frac{1}{w}) \tag{11}$$

In this scenario, the entropy is computed as:

$$Q(w) = \sum_{i=1}^{n} w_i \phi(w_i) = \sum_{i=1}^{n} w_i \log \left(\frac{1}{w_i} \right) \tag{12}$$

The maximum value of the Entropy is given by:

$$Q(\frac{1}{n}, \frac{1}{n}, .., \frac{1}{n}) = \sum_{i=1}^{n} \frac{1}{n} log(n) = log(n) = \phi(\frac{1}{n}) \tag{13}$$

and it suggests that the distribution is perfectly egalitarian.
Theil & Uribe (1967) was inspired by this concept when creating his index. He decides to apply it to an income distribution by applying two modifications. The first consists of replacing probabilities with income shares. The second considers the difference between the max value of entropy and the quantity Q(s) that corresponds to the value of disposable income at that point in the distribution, where s is the income vector of the distribution.

$$T = Q(\frac{1}{n}, \frac{1}{n}, .., \frac{1}{n}) - Q(s) = log(n) - Q(s) \tag{14}$$

Considering that:

$$Q(s) = \sum_{i=1}^{n} s_i log(\frac{1}{s_i}) = -\sum_{i=1}^{n} s_i log(s_i) \tag{15}$$

and

$$log(n) = \sum_{i=1}^{n} s_i log(n) \tag{16}$$

Then,

$$\begin{aligned} T &= log(n) - Q(s) \\ T &= \sum_{i=1}^{n} s_i log(n) s_i \end{aligned} \tag{17}$$

where

$$s_i = \frac{y_i}{n\mu} \tag{18}$$

and so the Theil index can be written as:

$$T = \sum_{i=1}^{n} \frac{y_i}{n\mu} log(n) \frac{y_i}{n\mu}$$

$$T = \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{\mu} log(n) \frac{y_i}{\mu} \tag{19}$$

where $\frac{y_i}{\mu}$ is the slope of the Lorenz curve at the quantile corresponding to y_i.

The index varies from 0 to 1, denoting perfect inequality at 0. Moreover, the Theil Index satisfies the properties of normalization, symmetry, population replicability, differentiability, scale independence, transferability and additive decomposition. Concerning the latter property, it can be shown that:

$$T = \frac{1}{n} \sum_{g=1}^{G} \sum_{i=1}^{n_g} \frac{y_i^g}{\mu} log(\frac{y_i^g}{\mu}) = \sum_{g=1}^{G} \sum_{i=1}^{n_g} \frac{y_i^g}{n\mu} log(\frac{y_i^g}{\mu_g} \frac{\mu_g}{\mu}) =$$

$$= \sum_{g=1}^{G} \sum_{i=1}^{n_g} \frac{y_i^g}{n\mu} \left(\log \frac{y_i^g}{\mu_g} + \frac{\mu_g}{\mu} \right) =$$

$$= \sum_{g=1}^{G} \left[\sum_{i=1}^{n_g} \frac{y_i^g}{n\mu} + \sum_{i=1}^{n_g} \frac{y_i^g}{n\mu} \log \left(\frac{\mu_g}{\mu} \right) \right] =$$

$$= \sum_{g=1}^{G} \frac{n_g \mu_g}{n\mu} \sum_{i=1}^{n_g} \frac{y_i^g}{n_g \mu_g} log \frac{y_i^g}{\mu_g} + \sum_{g=1}^{G} \frac{n_g \mu_g}{n\mu} log(\frac{\mu_g}{\mu}) \tag{20}$$

In equation 20, $\sum_{i=1}^{n_g} \frac{y_i^g}{n_g \mu_g} log \frac{y_i^g}{\mu_g} = T_g$ corresponds to the weighted sum of the Theil indices within the groups. Then, we can write the Theil Index as:

$$T = \sum_{g=1}^{G} \frac{n_g \mu_g}{n\mu} T_g + \sum_{g=1}^{G} \frac{n_g \mu_g}{n\mu} log(\frac{\mu_g}{\mu}) \tag{21}$$

It is possible to decompose the Theil index into a between part and a within one.

$$T = T_B + T_W$$

$$T_B = \sum_{g=1}^{G} \frac{n_g \mu_g}{n\mu} log(\frac{\mu_g}{\mu}) \tag{22}$$

$$T_W = \sum_{g=1}^{G} \frac{n_g \mu_g}{n\mu} T_g \tag{23}$$

In this paper, the Theil index is used to understand the inequality between different Italian regions concerning different levels of secondary school efficiency. Indeed, after calculating the conditional efficiency coefficients, the first phase of the paper applying the DEA Approach measured the average of the two conditional coefficients, and the vector was thus divided into percentiles.

3 Data and empirical analysis

The Italian school system comprises ten years of compulsory education, including elementary, middle, and high schools. Secondary schools, mandatory only up to the second year, are organized into Liceo and Technical Institutes. The former focuses on teaching predominantly humanities and scientific subjects, while the latter emphasizes the development of skills immediately applicable in the job market. During the academic year, students are tested orally and in writing to assess their preparation and competence in the various school subjects. Teachers evaluate students based on their learning and behavior, conducting midterm, periodic, and final assessments in line with the learning objectives outlined by the Ministry of Education. By the end of January, teachers issue report cards, that reflect the students' preparation. In the spring of the same academic year, students also participate in standardized tests. The National Institute prepares these tests for the Evaluation of the Educational System - Istituto Nazionale per la Valutazione del Sistema Educativo (INVALSI) - a research body supervised by the Ministry of Education.
Standardized tests are compulsory and are the same for all students. The Invalsi are administered at a national level and used to verify the level of competence of the respondents on a national scale. These tests present the same questions for all students and schools and often offer the same questions as standardized tests in other OECD countries.

The dataset implemented in this paper is extrapolated from the INVALSI database and refers to a 2022 survey on the student population of Italian high school students. The dataset presents 4015 high schools distributed across the Italian territory, but the sample is reduced due to a lack of data.
The analysis is carried out on two different distributions. The first is related to the scores obtained on the Invalsi standardized tests; the second is based on the official marks students obtained in Italian and Math at the end of the first semester of the 2021-2022 school year.

Invalsi's rating ranges from 1 to 5, where 5 means the student is very competent. Conversely, the Official Grades range from 1 to 10, where 10 corresponds to the maximum. To understand how the two distributions differ, the Official Grades are rescaled according to the Invalsi scoring scale, and some descriptive statistics are computed, as shown in Table 1. On average, students perform better on the Official Grade distribution than on the Invalsi. The latter also shows greater variability in the results.

As can be seen in Figure 1, the Skewness values suggest a slight asymmetry in both distributions. The negative Skewness of the Official Grades indicates a slight left-tailed asymmetry, suggesting that some higher scores pull the distribution to the left. In contrast,

Table 1: Descriptive statistics on Avg. Italian and Math votes in the 2 distributions.

	Official Marks	**Invalsi Scores**
Mean	3.61	2.91
Variance	0.27	1.33
Skewness	-0.11	0.0439
Kurtosis	2.81	2

the positive Skewness of the Invalsi scores suggests a slight right-tailed Skewness, with some lower scores pulling the distribution to the right. Furthermore, the Kurtosis values reflect distributions with moderate peaks for both official grades and Invalsi scores, implying a relatively normal distribution with heavier tails than the standard normal distribution.

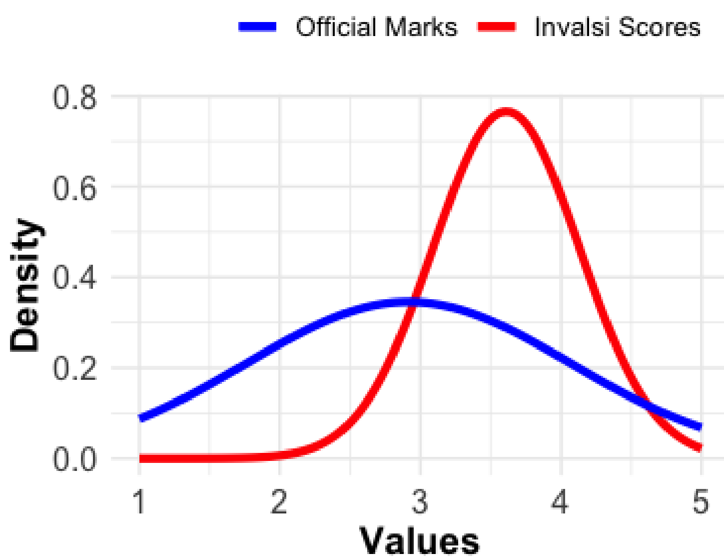

Figure 1: Official Marks and Invalsi Scores distributions

In the first stage of the analysis, particularly concerning the application of the DEA methodology (in both traditional and conditional versions)[3] to the DMUs (schools), constructing the efficient frontier involves the definition of inputs and outputs. For both distributions, the input considered is the same: *Study hours* (following De Witte & Kortelainen (2013); Lagravinese *et al.* (2020)), which corresponds to the number of hours spent at school by students. High schools in Italy have a time code decided at the ministerial level. The value of this variable is as follows: from 1 to 1.99 means from 20 to 24 hours per week spent at school; 2 to 2.99 means 25 to 29 hours; 3 to 3.99 means 30 to 34 hours; from. 4 to 4.99 means 35 to 39 hours, and finally, 5 means 40 hours or more.

[3]All the results are computed using di R package "rcDEA" Mergoni (n.d.)

The outputs considered in the Invalsi analysis are the scores obtained by students on the INVALSI standardized tests in Italian and Mathematics. Choosing educational achievements as outputs is in line with the economic literature [4]. The input and output variables are described in Table 2.

Table 2: Input and Output Variables - Invalsi scores

	Obs.	Mean	St. Dev	Min.	Max.
Input Study Hours	3012	3.06	0.37	1.00	5.00
Output Score Italian	3012	2.67	0.73	1.00	5.00
Output Score Math	3012	2.71	0.87	1.00	5.00

On the other hand, the outputs considered in the analysis of the Official Grades are precisely the average marks per school that are assigned by teachers to students at the end of the first semester in Italian and Mathematics. Table 3 describes the input and output variables considered in the Official Grades distribution.

Table 3: Input and Output Variables - Official Grades

	Obs.	Mean	St. Dev	Min.	Max.
Input Study Hours	3006	3.06	0.37	1.00	5.00
Output Mark Italian	3006	7.03	0.54	4.67	9.5
Output Mark Math	3006	6.65	0.57	3	9.5

For both distributions, the environmental variables are the same, and they are reported in Table 4. They are considered in the conditional DEA approach to understand which ones are more influential on a school's performance. Since the school system is a set of super-connected elements, it is essential to determine the most significant factors of their performance. Indeed, it includes students with their family backgrounds and abilities, teachers, knowledge, and so on.

Environmental variables:

- *School size*: the number of students per school, which determines the size of the school;
- *Liceo*: it is a dummy variable that assumes value one if high school is a Liceo and zero otherwise;
- *PC*: the number of computers per school;
- *Region*: the geographical area of the school.
- *% Female students.*: share of female students in the school.

[4] De Witte & Kortelainen (2013); Coco & Lagravinese (2014); Barra *et al.* (2015); Coco *et al.* (2020)

- *% Italian students*: share of students of Italian origin in the school;
- *% regular students*: share of students who have not failed in previous years.
- *ESCS*: index of the student's economic, social, and cultural condition;

Table 4: Environmental Variables

	Mean	St.Dev	Min.	Max.
Number of pc	43.34	37.00	0	435
School size	122.2	81.93	1.00	488.00
% Italian students	90.06	8.60	33.33	100.00
% regular students	78.51	19.39	1.79	100.00
% female students	48.83	20.72	0.65	100.00
ESCS	0.02	0.57	-2.81	1.78

Some clarifications must be made on environmental variables. In Italy, there are schools attended only by girls. As reported by Biemmi (2015), there is a robust phenomenon of gender segregation. The choice of high school is affected by a sexist division whereby males prefer to attend scientific or professional studies, while females tend to choose more humanistic studies.
Additionally, some schools have six or fewer students. This data is not only due to the lack of data but also to Italy's geographical position. There are some schools defined as insular or mountain, in which student attendance is minimal (Bandini (2019)). Furthermore, the ESCS is based on three indicators: (i) *HISEI*, the employment status of parents; (ii) *PARED*, the educational level of the parents expressed in years of formal education followed calculated according to international standards; (iii) *HOME POST*, the possession of some material goods understood as variables of proximity to an economic-cultural context favorable to learning. The calculation [5] of the ESCS is carried out using the analysis of the principal components of the three indicators introduced. In line with what is proposed by OECD-PISA, the factor scores associated with the first principal component (usually able to explain at least 50% of the total variance) are assumed as ESCS values. The ESCS by construction indicates zero mean and unit standard deviation. Therefore, a student with a strictly positive individual ESCS value is a student with a more favorable socioeconomic background than the Italian average. The ESCS index is widely used in literature because it is one of the most explicable variables of inequality in educational achievement, as also demonstrated by Lagravinese *et al.* (2020).

After measuring the efficiency for both distributions, the inequality of educational achievement at the regional level is measured through the application of Theil's index. Thanks to the property of decomposability, it is possible to understand the inequality in efficiency both within and between regions. The analysis was carried out on both the Invalsi and the Official distribution and then compared.

[5] https://www.editore.it/snv/allegati/01 A INVALSI escs slide.pdf

4 Results

4.1 The efficiency analysis

In this paper, the performance of Italian secondary schools is assessed by constructing a production function in which the input is the variable *study hours* and the outputs are, in a first analysis, the average scores per school obtained by students in the Invalsi standardized tests in Italian and Mathematics. In the second analysis, the Official Grades in both subjects are obtained in the first semester. The analysis is conducted on the dataset related to the Italian secondary schools in 2021-2022.
The results of the school efficiency analysis are obtained by implementing the DEA approach in both its versions, the traditional and the conditional. Subsequently, a Non-parametric Significance Test is computed through a regression. The dependent variable is the ratio obtained by relating the conditional coefficient to the unconditional one, and the regressors are all the environmental variables.

In Table 5, a comprehensive summary of efficiency coefficients obtained on the Invalsi distribution is presented. It describes the two distinct methodologies implemented to assess schools' performance. The first method gauges unconditioned performance, resulting in a mean efficiency coefficient of 1.85 and a standard deviation of 0.63. This approach provides a baseline evaluation of school efficiency, independent of environmental variables. Conversely, those factors are considered in the second methodology, where the mean efficiency coefficient is 1.38. The table shows that according to the traditional DEA, the number of efficient schools is 13, while the conditioned version suggests 999 efficient schools. This detailed examination underscores the importance of methodological considerations in evaluating school efficiency and highlights the diverse factors influencing educational outcomes in the assessed dataset.

Table 5: Summary of Efficiency Coefficient - Invalsi scores

	Obs.	Mean	St. Dev	Min.	Max.	$\Theta = 1$
Unconditional Performance	3012	1.85	0.63	1	4.75	13
Conditional Performance	3012	1.38	0.47	1	4.05	999

The analysis of the distribution of Official Grades, on the other hand, shows the results in Table 6. The table presents the traditional unconditional assessment and the conditional one. The first method has an average efficiency of 1.294 with a standard deviation of 0.095. This approach identifies just 7 schools as efficient according to the established criteria. In contrast, the second method, which conditions the performance assessment on environmental variables, generated a marginally lower average performance of 1.072, with a reduced standard deviation of 0.083. In this case, 1145 schools are identified as efficient.

As the comparison between the two distributions suggests, the Official Grades identifies more efficient schools than the Invalsi distribution according to the Conditional DEA Ap-

Table 6: Summary of Efficiency Coefficient - Official marks

	Obs.	Mean	St. Dev	Min.	Max.	$\Theta = 1$
Unconditional performance	3006	1.294	0.095	1	1.9	7
Conditional performance	3006	1.072	0.083	1	1.637	1145

proach.

To assess the impact of external factors on the performance of the Decision Making Unit (DMU), a Non-Parametric Significance Test is applied. This test involves a regression analysis where the dependent variable is the ratio between the conditioned efficiency coefficient and the unconditioned coefficient, calculated for each school. Various environmental factors represent the independent variables. The test aims to identify the average influence of each variable on DMU efficiency, emphasizing the direction of impact—whether positive or negative.
As shown in Table 7, the variables ESCS and school size appear to be highly significant for both distributions. Specifically, the ESCS variable exhibits a consistent positive influence on both measures of academic performance, suggesting that a higher socioeconomic background fosters academic achievement. However, school size demonstrates contrasting behavior. More populated schools correspond to increased performance in the distribution of Invalsi Scores, whereas smaller schools seem to enhance performance in the distribution of Official Grades. Additionally, the analysis of Invalsi Scores highlights the influence of school type: attending a Liceo increases the likelihood of achieving better efficiency. Conversely, schools attended predominantly by female students appear to reduce academic performance. It is important to note that in this specific analysis, the difference between scores in mathematics or Italian is not specified. This result is in line with the literature, in fact even Ricolfi (2023) points out that student performance in INVALSI tests differs between male and female students. The latter, in particular, report better results in Italian[6].
These findings contribute to understanding of the influence of school-related variables on the observed outcomes, emphasizing the importance of considering various contextual factors in educational assessments.

In the context of analyzing school efficiency, investigating the influence of school variables reveals key points.
Overall, the analysis indicates that the number of efficient Italian high schools tends to be higher when the conditional DEA Approach is applied.
Moreover, the application of conditional DEA suggests that the number of efficient schools is higher in the distribution of Official Grades rather than Invalsi Scores. However, the result is reversed with unconditional DEA.

[6]The table does not include the specific values of the Regional variables. However, the analysis suggests varying contributions from different regions, with certain regions displaying statistically significant effects while others indicate no significant impact. Calabria, Campania, Lazio, and Puglia are the schools with the worst school performance.

Table 7: Non-Parametric Significance Test

Variable	Invalsi Scores	Official Grades
Liceo	0.0236**	0.0032
	(0.0085)	(0.0029)
ESCS	0.0091***	0.0020***
	(0.0015)	(0.0005)
N. computers	0.0005	-0.0007
	(0.0023)	(0.0008)
School size	0.0135***	-0.0042***
	(0.0032)	(0.0011)
% Italian students	0.0604	-0.0034
	(0.0325)	(0.0111)
% regular students	-0.0068	0.0050
	(0.0085)	(0.0029)
% female students	-0.0310***	0.0021
	(0.0050)	(0.0017)

4.2 The schools' performance inequality at the regional level

Taking the results from the DEA Approach on the schools' performance, a more detailed analysis was conducted to examine the geographical distribution of these identified efficient schools across Italy. Once again, the two different distributions are taken into account: Invalsi Scores and Official Grades.

For each distribution, the coefficients of conditioned efficiency are presented in Table 8. Here, each row represents a distinct region in Italy, detailing the total number of schools, along with the count of schools deemed efficient within each assessment framework. For instance, regions like Lombardia and Veneto are the most school-populated regions, and the conditional DEA Approach suggests that schools using the Invalsi distribution are more efficient than the official one. On the other hand, regions such as Basilicata and Bolzano exhibit relatively lower total school counts, with fewer schools identified as efficient, particularly in the Official Grades assessment. This table provides a comparative overview, highlighting regional disparities in school efficiency across different assessment methodologies, contributing essential insights for potential educational policy interventions and resource allocation strategies.

To understand the efficiency differences among schools within the same region and across different regions, the Theil index is applied. In Table 9 it is possible to visualize a comparative analysis of inequality measured by the Theil Index between regions across educational achievements. The between-group inequality, highlighting disparities between regions, is notably lower in Official Grades compared to Invalsi Scores, indicating relatively less divergence in performance among regions in the Official Grades assessment. However, the average within-group inequality within regions shows a similar trend, with higher disparities in Invalsi Scores compared to Official Grades, signifying greater variance in achievement levels

Table 8: Number of Schools and Efficient Schools in Invalsi Scores and Official Grades by Region

Region	**Number of schools**	**Invalsi efficient schools**	**Official efficient schools**
Abruzzo	67	16	26
Basilicata	30	9	11
Calabria	115	17	35
Campania	378	73	104
Emilia Romagna	169	74	58
Friuli Venezia Giulia	50	27	15
Lazio	274	62	96
Liguria	66	24	23
Lombardia	548	261	221
Marche	97	30	33
Molise	21	6	7
Piemonte	177	80	63
Prov. Bolzano	13	5	6
Prov. Trento	31	13	20
Puglia	201	49	77
Sardegna	82	14	18
Sicilia	235	41	72
Toscana	157	56	69
Umbria	49	13	23
Valle d'Aosta	8	3	3
Veneto	238	113	99

within regions in the Invalsi Score assessment. Overall, the total inequality, encompassing both between-group and within-group disparities, is observed to be higher in Invalsi Scores than in Official Grades, suggesting a broader range of educational disparities across regions in the Invalsi Score assessment method.

Table 9: Comparison of Theil Index Inequality in Invalsi Score and Official Grades

	Invalsi Score	**Official Grades**
Between Inequality	0.014	0.0003
Avg. Within Inequality	0.038	0.003
Total inequality	0.052	0.003

The Theil index for within-region inequality in Invalsi Scores provides a detailed perspective of the educational landscape in the various Italian regions. Table 10 presents the values for each region, revealing the extent of disparities in average scores within each administrative division. Observing the data, regions like Valle d'Aosta, Piemonte, and Trentino-Alto Adige (Prov. Trento and Prov. Bolzano) demonstrate relatively lower within-region inequality in both Invalsi Scores and Official Grades, suggesting a more homogeneous educational achievement within these areas. Conversely, regions such as Sicilia, Campania, and Sardegna exhibit higher Theil Index values in both assessments, signifying greater disparities in educational outcomes within these regions.

Table 10: Theil Index - Within Regions Inequality in Invalsi Score and Official Grades

Region	**Invalsi Scores**	**Official Grades**
Abruzzo	0.0501	0.0025
Basilicata	0.0420	0.0017
Calabria	0.0523	0.0034
Campania	0.0664	0.0041
Emilia Romagna	0.0323	0.0022
Friuli Venezia Giulia	0.0307	0.0026
Lazio	0.0484	0.0024
Liguria	0.0329	0.0032
Lombardia	0.0255	0.0025
Marche	0.0490	0.0019
Molise	0.0226	0.0027
Piemonte	0.0252	0.0020
Prov. Bolzano	0.0473	0.0021
Prov. Trento	0.0219	0.0017
Puglia	0.0496	0.0027
Sardegna	0.0513	0.0036
Sicilia	0.0579	0.0038
Toscana	0.0248	0.0022
Umbria	0.0427	0.0016
Valle d'Aosta	0.0092	0.0038
Veneto	0.0190	0.0021

Figure 2 depicts the educational distribution across the 21 Italian regions based on the efficiency coefficient percentiles. It showcases the performance intervals—ranging from 0-2, which represents non-efficient schools, to 8-10, signifying efficient schools, for both Invalsi Scores and Official Grades. Each region's educational landscape is described in these intervals, revealing the percentages of schools in each percentile within every region.
The northern regions, such as Friuli Venezia Giulia, Lombardy, Veneto, Emilia Romagna, and Piemonte, emerge as those with the highest proportion of schools classified as efficient according to the Invalsi distribution. Specifically, Friuli Venezia Giulia stands out for the highest number of absolutely efficient schools in the Invalsi distribution. Conversely, for the Official Grades, the regions with the highest number of efficient schools are Veneto, Trentino Alto Adige, Umbria, Toscana and Lombardia.

All the most efficient schools are concentrated in the northern regions, highlighting a marked geographical division in the Italian educational landscape. In contrast, the southern regions take the lead in having the least efficient schools, particularly Calabria, Sicily, Sar-

dinia, and Campania, as revealed by the Invalsi distribution. Regarding the Official Grades distribution, the regions with the least efficient schools are Sardinia, Campania, and the only exception is Valle d'Aosta.
These results underscore the relevance of regional differences in the Italian educational system, emphasizing the urgency of targeted policies to address these inequalities and promote a fair distribution of school efficiency throughout the national territory.

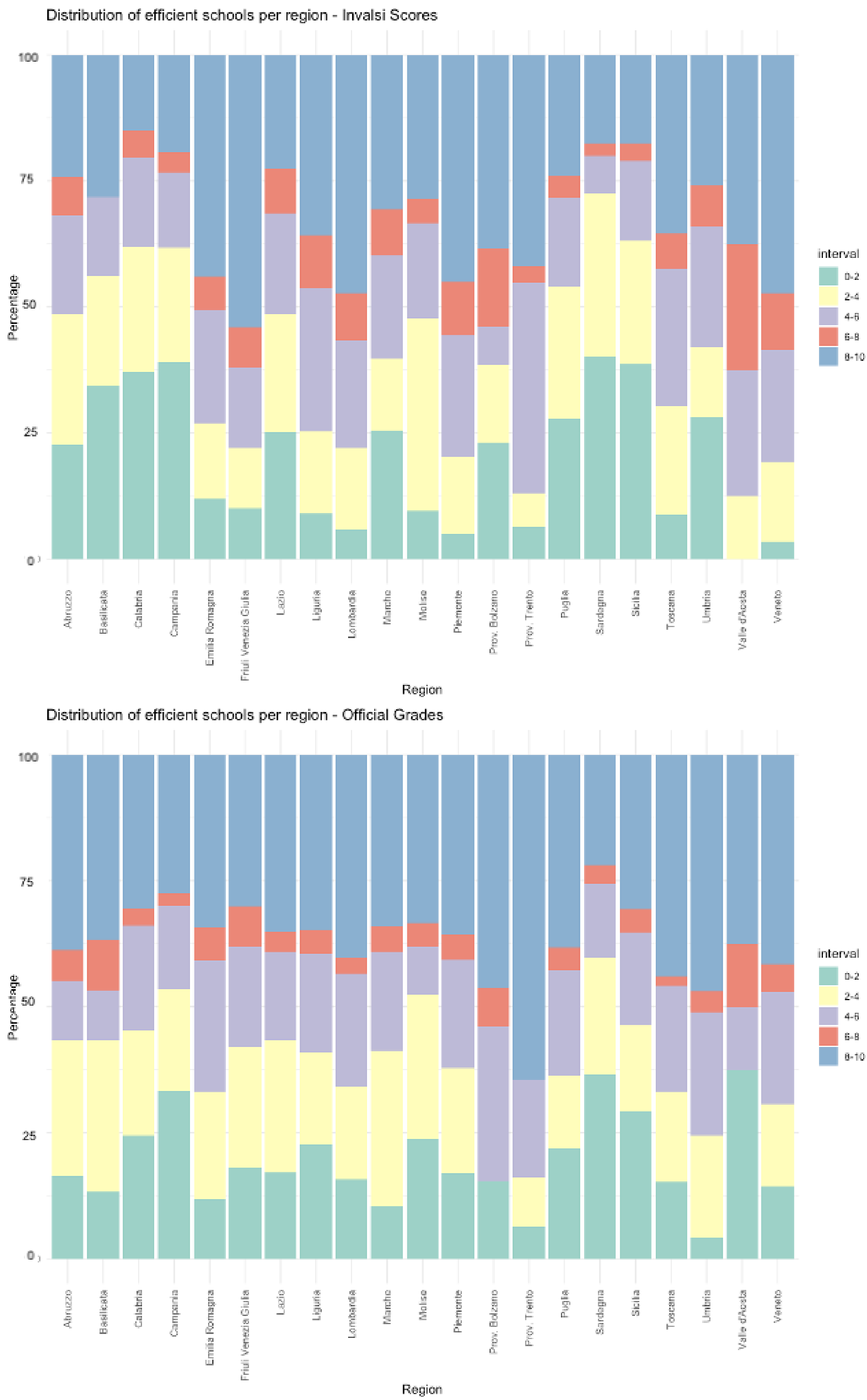

Figure 2: Share of Schools in the Rank Distribution across Regions in Italy in 2022

5 Conclusion

This paper investigates the intricate interplay between school performance and consequent inequality within the Italian secondary education system. The aim of the paper is to assess school performance by computing efficiency coefficients and exploring regional inequalities within educational achievements.

This analysis involves the examination of two distinct distributions: *Invalsi Scores* and *Official Grades* in Italian and Math. In the first part of the analysis, efficiency is measured through the application of the non-parametric, deterministic DEA Approach. In which a production function is constructed by relating input and output. Study hours are considered as the only input. Instead, as output: Invalsi Scores for the initial analysis and Official Grades for subsequent assessment. The DEA methodology is applied in both its traditional and conditional versions. The latter consists of conditioning to the environmental variables. The analysis showed that the variable Liceo emerged as a crucial factor significantly affecting the school's efficiency in the distribution of Invalsi Scores, in contrast to its seemingly insignificant influence on Official Grades distribution. Furthermore, both analyses emphasize the substantial influence of factors such as school size and socioeconomic status on school performance. These results illuminate the intricate web of factors influencing school efficiency and highlight the different determinants affecting school performance across different assessment metrics within the Italian secondary education system.

To assess the efficiency inequality among schools at the regional level, the Theil index was applied. The analysis highlights regional disparities in the efficiency of Italian schools based on Invalsi scores and Official Grades. Northern regions — Friuli Venezia Giulia, Lombardy, Veneto, Emilia Romagna and Piemonte — show a higher prevalence of schools classified as efficient, notably Friuli Venezia Giulia for absolute efficiency in Invalsi scores. Conversely, the southern regions, particularly Calabria, Sicily, Sardinia, and Campania, demonstrate more schools classified as less efficient in Invalsi scores. The same trend is replicated in the Official Grades distribution for Lombardia and Veneto. Interestingly, the Trentino Alto Adige, Umbria and Toscana perform better. These findings highlight the need for targeted policies to address educational inequalities and ensure a fair distribution of school efficiency across Italy.

The policy implications derived from this study underscore crucial points for addressing disparities within the Italian secondary education system. First, targeted interventions aimed at mitigating regional disparities are imperative, involving strategic investments in educational infrastructure, resource allocation, and tailored programs to uplift educationally disadvantaged regions. Second, gender-based initiatives are critical, necessitating the implementation of gender-sensitive educational strategies, mentorship programs, and the creation of inclusive learning environments to address gender-based disparities in Official Grade distribution. Third, recognizing the substantial influence of school-related factors on efficiency, strategies focusing on enhancing school efficiency—such as optimizing school size, addressing socio-economic disparities, and improving teaching quality—can foster a more equitable educational landscape.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

Table 11: Regression considering Score Ita as the dependent variable

	Coefficient	P-value
Intercept	2.22922	$2e-16$ ***
Study Hours	0.14195	$8.63e-05$ ***

Table 12: Regression considering Score Math as the dependent variable

	Coefficient	P-value
Intercept	2.50006	$< 2e-16$ ***
Study Hours	0.06349	0.144

Table 13: Regression considering Mean Score between Ita and Math as the dependent variable

	Coefficient	P-value
Intercept	2.36464	$< 2e-16$ ***
Study Hours	0.10272	0.0066 **

Table 14: Distribution of schools by regions and percentiles

Region	Total schools	Percentile	Schools in percentile	%
Abruzzo	67	0-2	14	20.89%
Abruzzo	67	2-4	13	19.40%
Abruzzo	67	4-6	13	19.40%
Abruzzo	67	6-8	13	19.40%
Abruzzo	67	8-10	14	20.89%
Basilicata	32	0-2	7	21.87%
Basilicata	32	2-4	6	18.75%
Basilicata	32	4-6	6	18.75%
Basilicata	32	6-8	6	18.75%
Basilicata	32	8-10	7	21.87%
Calabria	115	0-2	23	20.00%
Calabria	115	2-4	23	20.00%
Calabria	115	4-6	23	20.00%
Calabria	115	6-8	23	20.00%
Calabria	115	8-10	23	20.00%
Campania	387	0-2	78	20.15%
Campania	387	2-4	78	20.15%

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Table 14 – Continued from previous page

Region	Total schools	Percentile	Schools in percentile	%
Campania	387	4-6	76	19.63%
Campania	387	6-8	77	19.90%
Campania	387	8-10	78	20.15%
Emilia Romagna	169	0-2	34	20.11%
Emilia Romagna	169	2-4	34	20.11%
Emilia Romagna	169	4-6	33	19.52%
Emilia Romagna	169	6-8	34	20.11%
Emilia Romagna	169	8-10	34	20.11%
Friuli Venezia Giulia	50	0-2	10	20.00%
Friuli Venezia Giulia	50	2-4	10	20.00%
Friuli Venezia Giulia	50	4-6	10	20.00%
Friuli Venezia Giulia	50	6-8	10	20.00%
Friuli Venezia Giulia	50	8-10	10	20.00%
Lazio	279	0-2	56	20.07%
Lazio	279	2-4	56	20.07%
Lazio	279	4-6	55	19.71%
Lazio	279	6-8	56	20.07%
Lazio	279	8-10	56	20.07%
Liguria	67	0-2	14	20.89%
Liguria	67	2-4	13	19.40%
Liguria	67	4-6	13	19.40%
Liguria	67	6-8	13	19.40%
Liguria	67	8-10	14	20.89%
Lombardia	554	0-2	111	20.03%
Lombardia	554	2-4	111	20.03%
Lombardia	554	4-6	111	20.03%

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Table 14 – Continued from previous page

Region	Total schools	Percentile	Schools in percentile	%
Lombardia	554	6-8	112	20.21%
Lombardia	554	8-10	109	19.67%
Marche	98	0-2	22	22.44%
Marche	98	2-4	17	17.34%
Marche	98	4-6	20	20.40%
Marche	98	6-8	19	19.38%
Marche	98	8-10	20	20.40%
Molise	21	0-2	5	23.80%
Molise	21	2-4	4	19.04%
Molise	21	4-6	4	19.04%
Molise	21	6-8	4	19.04%
Molise	21	8-10	4	19.04%
Piemonte	179	0-2	36	20.11%
Piemonte	179	2-4	36	20.11%
Piemonte	179	4-6	35	19.55%
Piemonte	179	6-8	36	20.11%
Piemonte	179	8-10	36	20.11%
Prov. Bolzano	13	0-2	3	23.07%
Prov. Bolzano	13	2-4	2	15.38%
Prov. Bolzano	13	4-6	3	23.07%
Prov. Bolzano	13	6-8	2	15.38%
Prov. Bolzano	13	8-10	3	23.07%
Prov. Trento	31	0-2	7	22.58%
Prov. Trento	31	2-4	6	19.35%
Prov. Trento	31	4-6	6	19.35%
Prov. Trento	31	6-8	6	19.35%
Prov. Trento	31	8-10	6	19.35%
Puglia	206	0-2	42	20.30%
Puglia	206	2-4	41	19.90%
Puglia	206	4-6	41	19.90%
Puglia	206	6-8	41	19.90%

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Table 14 – Continued from previous page

Region	Total schools	Percentile	Schools in percentile	%
Puglia	206	8-10	41	19.90%
Sardegna	84	0-2	17	20.23%
Sardegna	84	2-4	17	20.23%
Sardegna	84	4-6	16	19.04%
Sardegna	84	6-8	17	20.23%
Sardegna	84	8-10	17	20.23%
Sicilia	238	0-2	48	20.16%
Sicilia	238	2-4	47	19.74%
Sicilia	238	4-6	48	20.16%
Sicilia	238	6-8	47	19.79%
Sicilia	238	8-10	48	20.16%
Toscana	161	0-2	33	20.49%
Toscana	161	2-4	32	19.87%
Toscana	161	4-6	32	19.87%
Toscana	161	6-8	32	19.87%
Toscana	161	8-10	32	19.87%
Umbria	50	0-2	10	20.00%
Umbria	50	2-4	10	20.00%
Umbria	50	4-6	10	20.00%
Umbria	50	6-8	10	20.00%
Umbria	50	8-10	10	20.00%
Valle d'Aosta	8	0-2	2	25.00%
Valle d'Aosta	8	2-4	1	12.50%
Valle d'Aosta	8	4-6	2	25.00%
Valle d'Aosta	8	6-8	1	12.50%
Valle d'Aosta	8	8-10	2	25.00%
Veneto	240	0-2	48	20.00%
Veneto	240	2-4	48	20.00%
Veneto	240	4-6	50	20.83%
Veneto	240	6-8	46	19.16%
Veneto	240	8-10	48	20.00%